The University of the State of New York

303b High School Examination

TENTH YEAR MATHEMATICS

Monday, June 21, 1948 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in tenth year mathematics, (*c*) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year.

Part II

Answer two questions from part II.

26 Prove that tangents drawn to a circle from an external point are equal. [10]

27 In a circle whose center is O, AB is a diameter and AE a chord. From any point C on chord AE, CD is drawn perpendicular to AB. Prove that $AB \times AD = AC \times AE$. [10]

28 CD is the median to hypotenuse AB of right triangle ABC. If CD is extended its own length to E and EA and EB are drawn, prove that $AEBC$ is a rectangle. [10]

29 Prove that the area of a trapezoid is equal to one half the product of the altitude and the sum of the bases. [10]

Part III

Answer two questions from part III.

30 The area of a parallelogram is 60 sq. in. The longer side is 12 in. and one of the angles of the parallelogram is 54°. Find to the *nearest tenth of an inch* the shorter side of the parallelogram. [10]

31 *a* A circle whose center is (4, 3) passes through the point (8, 6). Find the radius of the circle. [4]

b On the circle given in *a*, there is another point whose abscissa is 8. Find its ordinate. [4]

c Find the distance of the center from the origin. [2]

32 The vertices of a triangle are A (3, 2), B (8, 2) and C (5, 8). Find the length of the median from B to AC. [10]

*33 Using the formula for the slope of a line, show that

a A (2, 2), B (10, 4), C (13, 10) and D (5, 8) are the vertices of a parallelogram. [5]

b If the vertices of a quadrilateral are A (2, 0), B (6, 3), C (6, 8) and D (2, 5), the diagonals of the quadrilateral are perpendicular to each other. [5]

* This question is based on one of the optional topics in the syllabus.

Part IV

Answer one question from part IV.

34 *a* Prove that any point on the bisector of an angle is equidistant from the sides of the angle. [4]

b State and prove the converse of *a*. [2, 3]

c Is either the theorem in *a* or the converse stated in answer to *b*, taken alone, sufficient for proving the statement: The locus of points equidistant from the sides of an angle is the angle bisector? [Answer *yes* or *no*.] [1]

35 Read carefully statements *a* to *d* and in each case tell whether the reasoning is *sound* or *unsound*.

a It has been proved that a diagonal divides a parallelogram into two congruent triangles; it follows, therefore, that if a diagonal divides a quadrilateral into two congruent triangles, the quadrilateral is a parallelogram. [1]

b Rectangles and rhombuses are both parallelograms and since it has been proved that the diagonals of a rectangle are equal, it follows that the diagonals of a rhombus are equal. [1]

c If it is proved that angle A is not greater than angle B and that angle A is not less than angle B, it follows that angle A is equal to angle B. [1]

d Since it has been proved that the line joining the mid-points of two sides of a triangle is parallel to the third side, it follows that if a line does not join the mid-points of two sides of a triangle, it is not parallel to the third side. [1]

Each of the conclusions found in x, y, z below is reached by a type of reasoning similar to one of the types of reasoning used in *a*, *b*, *c* and *d* above. Indicate the statement, *a*, *b*, *c* or *d*, to which the reasoning in *each* case, x, y and z, corresponds most closely.

x Since the League of Nations failed, the United Nations will also fail. [2]

y In the month of July, it is warm; therefore, if it is May, it is cool. [2]

z All democracies have elective legislative assemblies. Since country A has an elective legislative assembly, it is a democracy. [2]

Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in part I. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 The hypotenuse of a right triangle is 17 and one leg is 15. Find the other leg. 1.....

2 The hypotenuse AB of right triangle ABC is twice leg BC. Find the number of degrees in angle ABC. 2.....

3 In right triangle ABC, altitude CD is drawn on the hypotenuse. If $CD = 6$ and $AD = 3$, find DB. 3.....

4 Find the altitude of an equilateral triangle whose side is 4. [Answer may be left in radical form.] 4.....

5 In triangle ABC, angle $C = 90°$, $\tan A = .3$ and $AC = 20$. Find BC. 5.....

6 An angle formed by a tangent and a secant is 75°. If the greater intercepted arc is 180°, find the number of degrees in the smaller intercepted arc. 6.....

7 Two parallel lines are cut by a transversal so that two interior angles on the same side of the transversal are represented by x and $2x + 30°$. Find the number of degrees in the smaller of these two angles. 7.....

8 M is the mid-point of line segment AB. The coordinates of point A are (6, 2) and of point M (6, 8). Find the coordinates of point B. 8.....

9 If two chords intersect within a circle in such a way that the intercepted arcs, taken in order, are x, $4x$, $5x$ and 60°, find the number of degrees in one of the angles at the point of intersection. 9.....

10 Find the area of a trapezoid if the altitude is 10 and the bases are 6 and 8. 10.....

11 Find the side of a square that is equal in area to a triangle whose base is 36 and whose altitude is 8. 11.....

12 Corresponding sides of two similar triangles are in the ratio 2:3. Find the ratio of the area of the smaller triangle to the area of the larger triangle. 12.....

13 Two similar polygons have 2 and 3 as a pair of corresponding sides. Find the ratio of the perimeter of the smaller polygon to the perimeter of the larger polygon. 13.....

14 Write the equation of the line which is the locus of all points equidistant from points (0, 12) and (0, 4). 14.....

15 Find the length of an arc of a circle if the arc is 30° and the radius is 12. [Answer may be left in terms of π.] 15.....

Directions (questions 16–18) — Indicate the correct answer to *each* question by writing on the line at the right the letter a, b or c.

16 In triangle ABC, angle A is greater than angle B and the bisectors of angle A and angle B meet in D. Then BD is (a) greater than AD (b) equal to AD (c) less than AD 16.....

17 If $x + 2y = 6$ and $2x - y = 2$, then the values of x and y are (a) underdetermined (b) determined (c) overdetermined 17.....

18 If the sum of two exterior angles of a triangle is 270°, the triangle is (a) acute (b) right (c) obtuse 18.....

Directions (questions 19–23) — In *each* of the following, if the statement is *always* true, write the word *true* on the line at the right; if it is *not always* true, write the word *false*.

19 The area of a regular polygon is equal to one half the product of its perimeter and its apothem. 19.....

20 If chord AB of a circle bisects chord CD, then AB is perpendicular to CD. 20.....

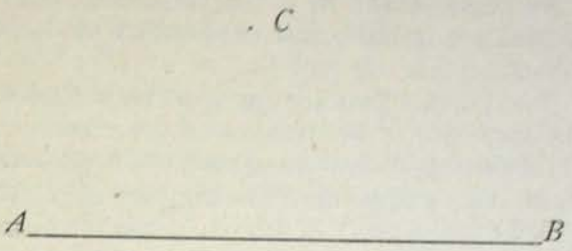
21 The altitudes of a triangle meet at a point which is inside the triangle. 21.....

22 A median of an equilateral triangle is equal to an altitude of the triangle. 22.....

23 The diagonals of a rhombus are equal. 23.....

Directions (questions 24–25) — Leave all construction lines on your paper.

24 From point C, construct a line perpendicular to AB.



25 Divide line segment AB into three equal parts.