

Examination January, 1979

Tenth Year Mathematics

PART ONE Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form. Write your answers in the spaces provided.

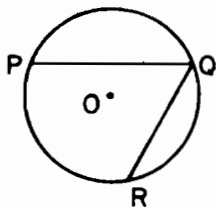
1. If one of the legs of a right triangle has length 5 and the hypotenuse has length 13, what is the length of the other leg? 1 _____

2. In rhombus $ABCD$, $AB = 3x + 12$ and $BC = 5x$. What is the value of x ? 2 _____

3. The circumferences of two circles are 7π and 15π , respectively. What is the ratio of the radius of the smaller circle to that of the larger circle? 3 _____

4. In a right triangle, the measures of the acute angles are x degrees and $5x$ degrees. Find x . 4 _____

5. In the accompanying diagram, $\angle PQR$ is inscribed in circle O . If $m\widehat{PQ} = 130$ and $m\widehat{QR} = 90$, find $m\angle Q$.



6. Triangle ABC has a right angle at C . If median \overline{CD} is drawn and $AB = 10$, find CD . 6 _____

7. An exterior angle at the base of an isosceles triangle measures 100° . How many degrees are there in the measure of the vertex angle of the triangle? 7 _____

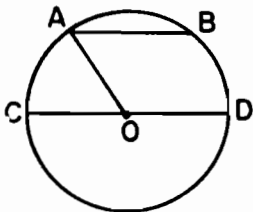
8. Find the number of degrees in the sum of the exterior angles of a quadrilateral. 8_____

9. The coordinates of two of the vertices of parallelogram $ABCD$ are $A(2,6)$ and $C(5,10)$. What is the length of diagonal \overline{AC} ? 9_____

10. The area of an equilateral triangle is $\frac{25\sqrt{3}}{4}$. Find the length of a side of the triangle. 10_____

11. Find the slope of the line that passes through the points $(-1,4)$ and $(5,9)$. 11_____

12. In the accompanying diagram, chord \overline{AB} is parallel to diameter \overline{CD} and radius \overline{OA} is drawn. If $m\widehat{AB} = 50$, find $m\angle AOC$.



12_____

13. The circumference of a circle is 18π centimeters. If the length of an arc of this circle is 3π centimeters, find the number of degrees in the measure of the arc. 13_____

14. In parallelogram $ABCD$, the bisectors of angles A and B intersect at E . Find $m\angle AEB$. 14_____

15. The lengths of the diagonals of a rhombus are 5 and 8. What is the area of the rhombus? 15_____

DIRECTIONS (16-29): Write in the space provided the numeral preceding the expression that best completes each statement or answers each question.

16. "If a student is in Homeroom 203, then she is a senior."
If this statement is true, then which must be true?

- (1) If a senior is not in Homeroom 203, she is not a senior.
 (2) If a student is not a senior, she is not in Homeroom 203.
 (3) Room 203 is the only senior homeroom.
 (4) Room 202 is not a senior homeroom.

16_____

17. The locus of points equidistant from two distinct points A and B is

- (1) one line (3) two lines
 (2) one circle (4) two circles

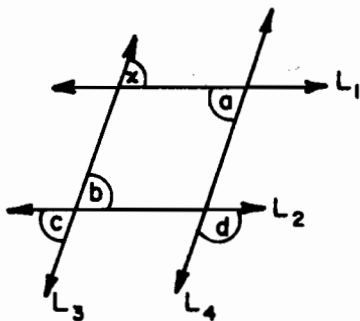
17_____

18. Which set could represent the lengths of the sides of a triangle?

- (1) $\{1, 3, 6\}$ (3) $\{2, 10, 12\}$
 (2) $\{2, 4, 7\}$ (4) $\{4, 6, 8\}$

18_____

19. In the accompanying diagram, if $L_1 \parallel L_2$ and $L_3 \parallel L_4$, then $\angle x$ is *not* always congruent to which angle?



- (1) $\angle a$ (2) $\angle b$ (3) $\angle c$ (4) $\angle d$ 19_____

20. In triangle ABC , $m\angle A = 30$, $m\angle C = 90$, and $BC = 10$. What is the length of \overline{AB} ?

- (1) 20 (2) $10\sqrt{3}$ (3) 10 (4) 5 20_____

21. In triangle ABC , $CB > CA$, D is a point on \overline{AC} , and E is a point on \overline{BC} such that $CE = CD$. Which statement is always true?

- (1) $m\angle CDE > m\angle A$ (3) $EB > AD$
 (2) $m\angle B = m\angle CED$ (4) $EB = AD$ 21_____

22. In a trapezoid, the length of the median is 14 and the length of one base is 10. The length of the other base is

- (1) 18 (2) 12 (3) 6 (4) 4 22_____

23. What are the coordinates of the midpoint of the line segment which joins the points whose coordinates are $(4, -1)$ and $(0, 2)$?

- (1) $2, 1/2$ (3) $(2, -3/2)$
 (2) $(2, -1/2)$ (4) $(-2, 3/2)$ 23_____

24. If two triangles have equal bases and the altitudes drawn to these bases are equal, then the two triangles are *always*

- (1) obtuse (3) congruent
 (2) equal in area (4) similar 24_____

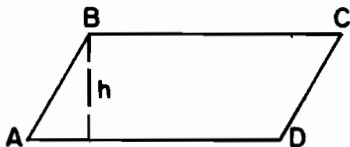
25. Which figures are *always* similar?

- (1) two rectangles (3) two hexagons
 (2) two parallelograms (4) two circles 25_____

26. The coordinates of point A are $(3, 0)$ and the coordinates of point B are $(7, 0)$. An equation of the locus of points equidistant from A and B is

- (1) $x = 10$ (2) $y = 10$ (3) $x = 5$ (4) $y = 5$ 26_____

27. As shown in the accompanying diagram, h is an altitude of parallelogram $ABCD$. Which equation can be used to find the length of h ?

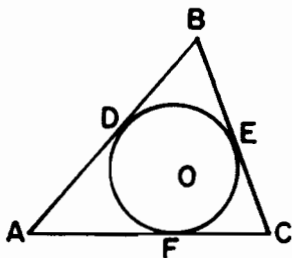


- (1) $h = AB \cos A$ (3) $h = AD \tan A$
 (2) $h = AB \sin A$ (4) $h = AD \cos A$ 27_____

28. In circle O , chords \overline{AB} and \overline{CD} intersect at E . If $AE = 2$, $EB = 8$, and $CE = 4$, then ED equals

- (1) 16 (2) 10 (3) 6 (4) 4 28_____

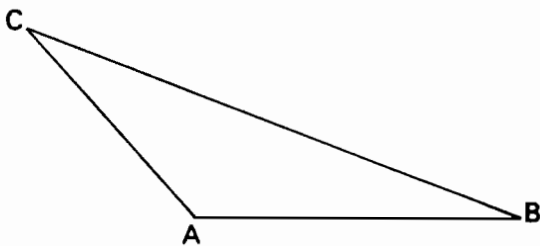
29. In the accompanying diagram, \overline{AB} , \overline{BC} , and \overline{AC} are tangent to circle O at points D , E , and F , respectively. If $AB = 14$, $BE = 6$, and $EC = 5$, then what is the length of \overline{AC} ?



- (1) 12 (2) 13 (3) 17 (4) 25 29_____

DIRECTIONS (30): Leave all construction lines on the answer sheet.

30. On the answer sheet, find by construction the midpoint of \overline{AB} and draw the median to side \overline{AB} in given triangle ABC . 30_____



PART TWO Answer four questions from this part. Show all work unless otherwise directed.

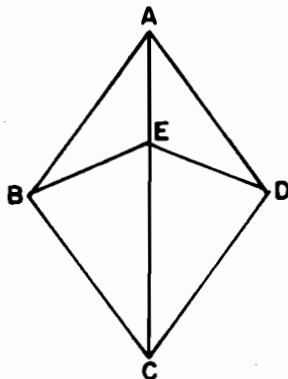
31. Prove either a or b , but not both.

- a Two right triangles are congruent if the hypotenuse and a leg of one are congruent to the corresponding parts of the other. [10]

OR

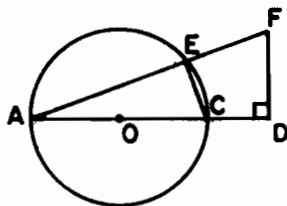
b The measure of an angle formed by two tangents is equal to one-half the difference of the measures of the intercepted arcs. [10]

32. Given: quadrilateral $ABCD$, diagonal \overline{AC} , $\overline{BE} \cong \overline{ED}$, $\overline{BC} \cong \overline{CD}$.



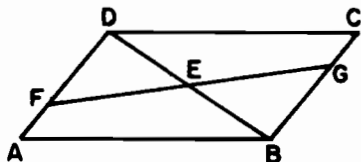
Prove: $\triangle AEB \cong \triangle AED$ [10]

33. In the accompanying diagram, diameter \overline{AC} of circle O is extended to point D , $\overline{DF} \perp \overline{AD}$, \overline{AEF} , where E is a point on circle O and \overline{EC} are drawn.



Prove: $AF \times EC = AC \times FD$ [10]

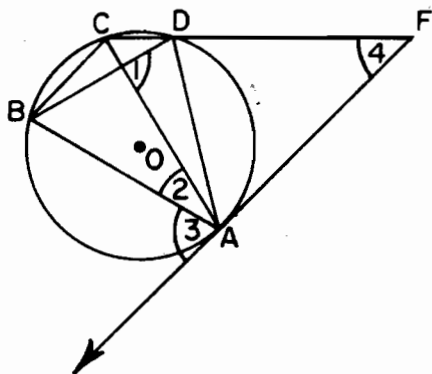
34. Given: parallelogram $ABCD$, \overline{FEG} , \overline{AFD} , \overline{BGC} , E is the midpoint of diagonal \overline{BD} .



Prove: $a \triangle FED \cong \triangle GEB$ [6]

b area of $ABEF =$ area of $CDEG$ [4]

35. Given: quadrilateral $ABCD$ inscribed in circle O , diagonals \overline{BD} and \overline{CA} , secant \overline{FDC} ; tangent \overline{FA} ; $m\widehat{BC}:m\widehat{CD}:m\widehat{DA}:m\widehat{AB} = 2:1:4:5$.



a Find $m\widehat{CD}$. [2]

b Find $m\angle 1$. [2]

c Find $m\angle 2$. [2]

d Find $m\angle 3$. [2]

e Find $m\angle 4$. [2]

36. Regular pentagon $ABCDE$ has a side of length 10. Find the length of an apothem of the pentagon to the nearest integer. [10]

37. The vertices of triangle ABC are $A(2,1)$, $B(4,7)$, and $C(8,3)$.

a Using the methods of coordinate geometry, show that $\triangle ABC$ is isosceles and state a reason for your conclusion. [5]

b Find the area of $\triangle ABC$. [5]