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The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

# TENTH YEAR MATHEMATICS

Wednesday, January 25, 1967 — 1:15 to 4:15 p.m., only

**The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.**

## Part I

**Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet.**

1 The measures of the angles of a triangle are in the ratio 1 : 4 : 5. Find the number of degrees in the smallest angle of the triangle.

2 In triangle $ABC$, angle $A = 40°$, angle $B = 70°$, and $AC = 5$ inches. Find the number of inches in the length of $AB$.

3 The ratio of corresponding apothems of two regular pentagons is 4 : 1. What is the ratio of the perimeter of the larger pentagon to the perimeter of the smaller?

4 Express in radical form the distance between the points whose coordinates are (—1,3) and (0,7).

5 Two parallel lines are cut by a transversal. A pair of interior angles on the same side of the transversal are represented by $(x + 25)°$ and $(3x + 15)°$. Find the number of degrees in the measure of the smaller angle.

6 Find the number of degrees in an exterior angle of a regular polygon with 12 sides.

7 If the length of a side of an equilateral triangle is 6, express the area of the triangle in radical form.

8 The base angles of an isosceles trapezoid are each 45° and the bases are 6 and 10. Find the length of the altitude of the trapezoid.

9 The legs of a right triangle measure 6 inches and 8 inches. Find the number of inches in the length of the radius of the circumscribed circle.

10 Chords $AB$ and $CD$ intersect inside a circle at point $E$. If $AE = 3$, $EB = 12$, and $CE = 9$, find the length of $ED$.

11 Find the area of a square whose diagonal is 8.

12 The coordinates of point $A$ are $(3a,b)$ and the coordinates of point $B$ are $(5a,3b)$. Find in terms of $a$ and $b$ the coordinates of the midpoint of the segment $AB$.

13 The length of a side of a rhombus is 5. If the length of one diagonal is 6, find the length of the other diagonal.

14 The diameter of a circle is 15 inches. If this diameter is extended 5 inches beyond the circle to point $A$, find the number of inches in the length of a tangent to the circle from point $A$.

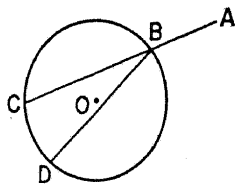
15 If the circumference of a circle is $10\pi$, what is the length of a side of a regular hexagon which is inscribed in this circle?

16 In $\triangle ABC$, angle $A = 52°$, $AC = 12$, and $AB = 8$. Find to the *nearest tenth* the length of the altitude from $B$ to $AC$.

17 Two tangents to a circle from an external point are perpendicular to each other. Find the number of degrees in the measure of the smaller of the two intercepted arcs of the circle.

18 An isosceles triangle has equal sides of length 8. Each base angle measures 30°. Express in radical form the length of the third side.

19 In the accompanying figure, $BD$ is a chord of circle $O$ and $ABC$ is a secant. If the measure of arc $DC$ is 50°, find the number of degrees in the measure of $\angle ABD$.




20 The radius of a circle is 4. Find in terms of $\pi$ the area of a sector of this circle whose angle is 45°.

21 In an isosceles right triangle, the length of one leg is 10. Express in radical form the length of the altitude upon the hypotenuse.

22 In triangle $ABC$, $D$ is a point on $AC$ and $E$ is a point on $BC$ such that $DE \parallel AB$. If $CD = 6$, $DA = 4$, and $EB = 3$, find the length of $EC$.

*Directions* (23–29): Write in the space provided on the separate answer sheet the *number* preceding the term that best completes the statement or answers the question.

23 The number of points equidistant from two parallel lines and also equidistant from two points on one of the lines is exactly

(1) 1
(2) 2
(3) 3
(4) 4

24 Which of the following represents a good *definition*?

(1) Adjacent angles are two angles with a common side.
(2) The hypotenuse is the longest side of a triangle.
(3) Similar polygons are polygons which have their corresponding angles equal and their corresponding sides proportional.
(4) Complementary angles are angles whose sum is a right angle.

25 If two sides of a triangle are 3 and 6, the length of the third side may be

(1) 1
(2) 6
(3) 3
(4) 9

26 If $PA$ and $PB$ are tangents drawn to circle $O$ from point $P$ and the number of degrees in the measure of angle $AOP$ is 40, then the number of degrees in the measure of angle $APB$ is

(1) 50
(2) 80
(3) 100
(4) 140

27 An equation of the circle whose center is at the origin and which passes through the point (2,0) is

(1) $x = 2$
(2) $x + y = 2$
(3) $x^2 + y^2 = 2$
(4) $x^2 + y^2 = 4$

28 The center of a circle inscribed in a triangle is located at the intersection of the

(1) angle bisectors of the angles of the triangle
(2) medians of the triangle
(3) altitudes of the triangle
(4) perpendicular bisectors of the sides of the triangle

29 Given the statement "If you have reached the age of 21 years, you are eligible to vote in New York State." Which is the inverse of this statement?

(1) If you have reached the age of 21, you are not eligible to vote in New York State.
(2) If you are eligible to vote in New York State, you have reached the age of 21.
(3) If you have not reached the age of 21, you are not eligible to vote in New York State.
(4) If you are not eligible to vote in New York State, you have not reached the age of 21.

*Directions* (30): Leave all construction lines on the answer sheet.

30 *On the answer sheet,* construct the bisector of angle $PAR$.

Answers to the following questions are to be written on paper supplied by the school.

## Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove *either* *a* or *b* but *not* both: [10]

*a* If two angles of a triangle are equal, the sides opposite these angles are equal.

*OR*

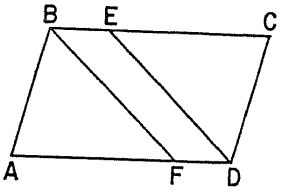
*b* The area of a parallelogram is equal to the product of one side and the altitude drawn to that side.

32 In the accompanying figure, $ABCD$ is a parallelogram with $DE$ and $BF$ bisecting angles $D$ and $B$, respectively.

Prove:

*a* $BF = DE$ [6]

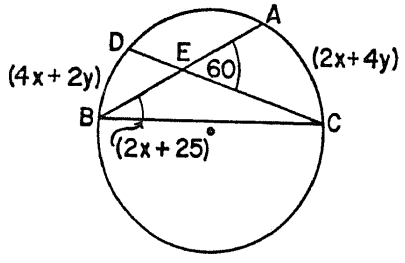
*b* $BEDF$ is a parallelogram [4]




33 In right triangle $ABC$, $CD$ is the altitude on hypotenuse $AB$ and $DE$ is perpendicular to $BC$.

Prove: $BC \times CD = AB \times DE$ [10]

34 In the accompanying figure, chords $AB$ and $CD$ intersect at point $E$ and chord $BC$ is drawn. The number of degrees in arcs $AC$ and $BD$ are represented by $(2x + 4y)$ and $(4x + 2y)$, respectively. The measure of angle $ABC$ in degrees is represented by $(2x + 25)$ and the measure of angle $AEC$ is 60°.




*a* In terms of $x$ and $y$, write a set of equations that can be used to solve for $x$ and $y$. [2, 2]

*b* Solve the set of equations written in answer to *a* to obtain the values of $x$ and $y$. [4]

*c* Find the number of degrees in the arcs $AC$ and $BD$. [2]

35 The coordinates of the vertices of $\triangle ABC$ are $A$ (1,—2), $B$ (2,5), and $C$ (—2,1).

*a* Find the lengths of $AB$, $BC$, and $AC$. [2, 2, 2]

*b* Show that triangle $ABC$ is a right triangle and write a reason for this conclusion. [2]

*c* Find the area of triangle $ABC$. [2]

36 If one side of a rhombus is 25 and the longer diagonal is 40, find the

*a* length of the shorter diagonal [3]

*b* area of the rhombus [2]

*c* length of the altitude [3]

*d* radius of the inscribed circle [2]

*37 Given the trapezoid $ABCD$ with the bases $AB$ and $DC$. The coordinates of the vertices are $A$ (0,0), $B$ ($k$,5), $C$ (7,—1), and $D$ ($k$,—3).

*a* Express the slope of $AB$ in terms of $k$. [2]

*b* Express the slope of $DC$ in terms of $k$. [2]

*c* Write an equation that can be used to solve for $k$ and solve this equation for $k$. [3]

*d* Using the value of $k$ obtained in answer to *c*, write an equation of the line through $C$ and $D$. [3]

* This question is based on an optional topic in the syllabus.

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Part I Score:.....

Rater's Initials:

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# TENTH YEAR MATHEMATICS

Wednesday, January 25, 1967 — 1:15 to 4:15 p.m., only

## ANSWER SHEET

Pupil.....Teacher.....

School.....

Name and author of textbook used.....

Your answers to Part I should be recorded on this answer sheet.

### Part I

**Answer all questions in this part.**

| | | |
|---|---|---|
| 1..... | 9..... | 17..... |
| 2..... | 10..... | 18..... |
| 3..... | 11..... | 19..... |
| 4..... | 12..... | 20..... |
| 5..... | 13..... | 21..... |
| 6..... | 14..... | 22..... |
| 7..... | 15..... | 23..... |
| 8..... | 16..... | 24..... |

Questions 25 through 30 should be answered on the back of this page.



[OVER]

Tear Here

25.....

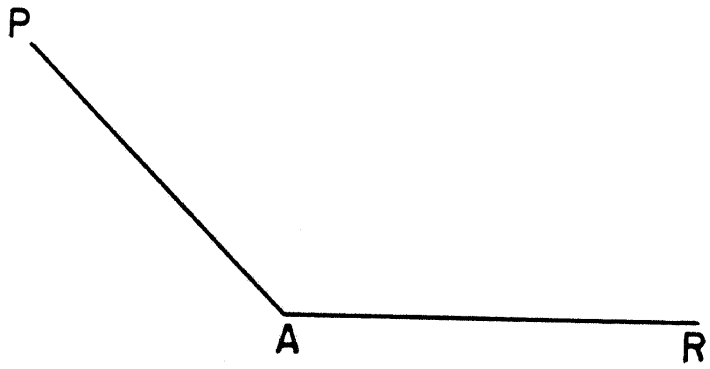
26.....

27.....

28.....

29.....

30



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# FOR TEACHERS ONLY

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## SCORING KEY

## TENTH YEAR MATHEMATICS

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

### Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 23–29, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3, or 4.

(1) 18

(2) 5

(3) 4:1

(4) $\sqrt{17}$

(5) 60

(6) 30

(7) $9\sqrt{3}$

(8) 2

(9) 5

(10) 4

(11) 32

(12) $(4a,2b)$

(13) 8

(14) 10

(15) 5

(16) 6.3

(17) 90

(18) $8\sqrt{3}$

(19) 155

(20) $2\pi$

(21) $5\sqrt{2}$

(22) $4\frac{1}{2}$ *or* $\frac{9}{2}$

(23) 1

(24) 3

(25) 2

(26) 3

(27) 4

(28) 1

(29) 3

[OVER]

## Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(34) *a* $2x + 25 = \frac{1}{2}(2x + 4y)$
$60 = \frac{1}{2}[(4x + 2y) + (2x + 4y)]$ [2, 2]

*b* $x = 5$
$y = 15$ [4]

*c* 70, 50 [2]

(35) *a* $5\sqrt{2}$, $4\sqrt{2}$, $3\sqrt{2}$ *or* $\sqrt{50}$, $\sqrt{32}$, $\sqrt{18}$ [2, 2, 2]

*b* A statement such as "If the square of the longest side of a triangle is equal to the sum of the squares of the other two sides, the triangle is a right triangle" would be acceptable. [2]

*c* 12 [2]

(36) *a* 30 [3]

*b* 600 [2]

*c* 24 [3]

*d* 12 [2]

(37) *a* $\frac{5}{k}$ [2]

*b* $\frac{2}{7 - k}$ [2]

*c* $\frac{5}{k} = \frac{2}{7 - k}$
$k = 5$ [3]

*d* $y = x - 8$ [3]