The University of the State of New York

10

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Tuesday, January 25, 1966 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet.

1 The angles of triangle ABC are in the ratio 1 : 5 : 9. Find the number of degrees in the smallest angle.

2 The sides of a triangle are 7, 10 and 13. Find the perimeter of the triangle formed by joining the midpoints of the sides of the triangle.

3 Two angles are supplementary and one angle is 30° more than twice the other. Find the number of degrees in the smaller angle.

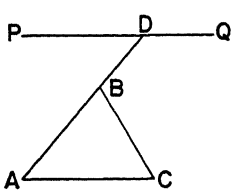
4 The area of a circle is 36π. Find the length of the diameter of this circle.

5 In trapezoid $ABCD$, bases AB and DC are 14 and 6, respectively. If the area is 40, find the length of the altitude of the trapezoid.

6 A chord 2 units long is drawn in a circle whose radius is 4 units. Find in radical form the distance of the chord from the center of the circle.

7 A circle whose center is at point A (1,—3) passes through point B (7,5). Find the length of the radius of this circle.

8 In the accompanying diagram, $PQ \parallel AC$ and AB extended meets PQ at D.



If angle $ABC = 70°$ and angle $PDB = 50°$, find the number of degrees in angle ACB.

9 One angle of a regular polygon is 144°. Find the number of sides of the polygon.

10 A tangent and a secant are drawn to a circle from an external point. The tangent is 10 inches long and the external segment of the secant is 5 inches long. Find the number of inches in the length of the secant.

11 A square is inscribed in a circle whose radius is 2 inches. Find the number of square inches in the area of the square.

12 If point B is located on line segment AC so that $\frac{AB}{AC} = \frac{3}{7}$, find the numerical value of $\frac{AB}{BC}$.

13 From point P outside a circle, tangent PA and secant PBC are drawn. If angle P contains 20° and arc AC is 100°, find the number of degrees in arc AB.

14 In right triangle ABC, CD is the altitude upon hypotenuse AB. If $AB = 10$ and $AD = 2$, find the length of altitude CD.

15 In triangle ABC, angle $C = 60°$ and $AC > BC$. Which angle of the triangle contains more than 60°?

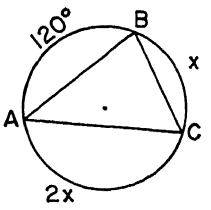
16 The radius of a circle is 12 inches. Find in terms of π the number of inches in the length of an arc intercepted by a central angle of 60°.

17 In circle O, two chords PQ and RS intersect at point T. If $RT = 2$, $PT = 4$ and $QT = 3$, find the length of ST.

18 The hypotenuse of a right triangle is 15 and a leg is 8. Find to the *nearest degree* the angle included between these two sides.

19 The area of an equilateral triangle is $9\sqrt{3}$. Find the length of a side of the triangle.

20 Triangle ABC is inscribed in a circle as shown in the accompanying figure. If $\overset{\frown}{AB} = 120°$, $\overset{\frown}{BC} = x°$ and $\overset{\frown}{CA} = 2x°$, find the number of degrees in angle A.



21 Find the coordinates of the midpoint of the line segment joining the points (1,—3) and (7,—3).

22 Corresponding sides of two similar triangles are in the ratio 1 : 3. If the area of the larger triangle is 18, find the area of the smaller triangle.

Directions (23–29): Write in the space provided on the separate answer sheet the *number* preceding the expression that best completes *each* statement or answers *each* question.

23 Given the line segment AB. The locus of the centers of all circles which have AB as a chord is
(1) one point
(2) two points
(3) a line
(4) a circle

24 The center of the circle inscribed in a triangle is the point of intersection of the
(1) angle bisectors
(2) altitudes
(3) perpendicular bisectors of the sides
(4) medians

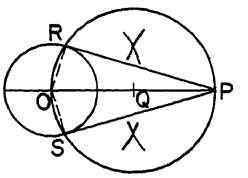
25 Which set of numbers can *not* be the lengths of the sides of a right triangle?
(1) 1, 1, $\sqrt{2}$
(2) 1, 1, $\sqrt{3}$
(3) 1, 2, $\sqrt{5}$
(4) 6, 8, 10

26 Which statement is a definition of a regular polygon?
(1) A regular polygon is a polygon which can be inscribed in a circle.
(2) A regular polygon is a polygon which is both equilateral and equiangular.
(3) A regular polygon is a polygon which has all its sides equal.
(4) A regular polygon is a polygon whose area is one-half the product of its perimeter and its apothem.

27 Given: If a quadrilateral is a parallelogram, the opposite sides are equal. Which statement concerning this theorem is true?
(1) The converse and the inverse are both true.
(2) The converse and the inverse are both false.
(3) The converse is true but the inverse is false.
(4) The converse is false but the inverse is true.

28 Which of the following is *not* sufficient to determine that a parallelogram is a rhombus?
(1) two adjacent sides equal
(2) perpendicular diagonals
(3) diagonals that bisect its angles
(4) diagonals equal

29 The accompanying figure shows the construction of the tangents to a circle from an external point P.



As point P approaches circle O, which angle will increase in measure?
(1) $\angle ORP$
(2) $\angle ROS$
(3) $\angle RPS$
(4) $\angle ROP$

Directions (30): Leave all construction lines on the answer sheet.

30 *On the answer sheet,* construct the altitude from A to side BC in triangle ABC.

Answers to the following questions are to be written on paper supplied by the school.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

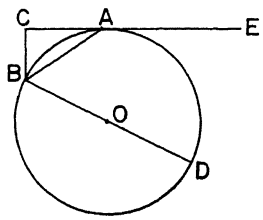
31 Prove *either a* or *b* but *not* both: [10]

a A diameter perpendicular to a chord of a circle bisects the chord and its arcs.

OR

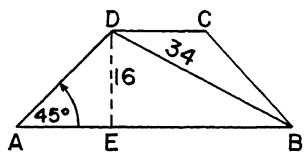
b The square of the hypotenuse of a right triangle is equal to the sum of the squares of the legs.

32 In the accompanying diagram, CE is tangent to circle O at point A.



BD is a diameter and AB is a chord of the circle. If $BC \perp CE$ at C, prove $(AB)^2 = BC \times BD$. [10]

33 In the accompanying figure, the bases of isosceles trapezoid $ABCD$ are AB and DC, with AB the longer base. Altitude $DE = 16$, diagonal $DB = 34$ and angle $A = 45°$.



a Find the number of units in the length of AE, EB, AB and DC. [1,3,1,3]

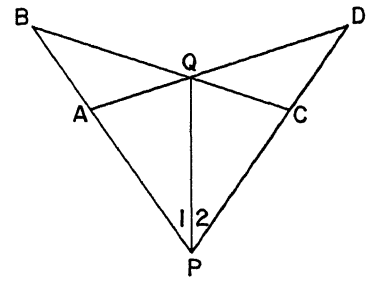
b Find the number of square units in the area of trapezoid $ABCD$. [2]

34 The side of a regular pentagon is 24.

a Find to the *nearest tenth* the apothem of the pentagon. [6]

b Using the answer found for part *a*, find to the *nearest square unit* the area of the pentagon. [4]

35 Given straight lines PQ, PAB, PCD, AQD and CQB. $\angle 1 = \angle 2$ and $AP = CP$.



Prove:

a $\triangle APQ \cong \triangle CPQ$ [3]

b $QB = QD$ [7]

36 The coordinates of the vertices of quadrilateral $ABCD$ are A (0,5), B (3,4), C (0,—5) and D (—3,—4).

a Using graph paper, draw quadrilateral $ABCD$. [1]

b Show that $ABCD$ is a parallelogram. [5]

c Show that $ABCD$ is a rectangle. [4]

*37 In triangle ABC, side AB lies on the line whose equation is $y = x + 3$ and side AC lies on the y-axis. The coordinates of M and N, the midpoints of sides AC and BC, are (0,—3) and $(2,k)$, respectively.

a Find the coordinates of A and of C. [1,1]

b Find the slope of AB. [2]

c Express the slope of MN in terms of k. [2]

d Find the value of k. [2]

e Write an equation of the line through M and N. [2]

* This question is based on an optional topic in the syllabus.

FOR TEACHERS ONLY

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SCORING KEY

TENTH YEAR MATHEMATICS

Tuesday, January 25, 1966 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 23–29, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) 12

(2) 15

(3) 50

(4) 12

(5) 4

(6) $\sqrt{15}$

(7) 10

(8) 60

(9) 10

(10) 20

(11) 8

(12) $\frac{3}{4}$

(13) 60

(14) 4

(15) B

(16) 4π

(17) 6

(18) 58

(19) 6

(20) 40

(21) (4,—3)

(22) 2

(23) 3

(24) 1

(25) 2

(26) 2

(27) 1

(28) 4

(29) 3

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(33) *a* 16, 30, 46, 14 [1,3,1,3]
b 480 [2]

(34) *a* 16.5 [6]
b 990 [4]

(37) *a* A (0,3), C (0,—9) [1,1]
b 1 [2]
c $\frac{k+3}{2}$ [2]
d —1 [2]
e $y = x - 3$ [2]