The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Monday, January 27, 1964 — 1:15 to 4:15 p.m., only

Name of pupil..... Name of school.....

Name and author of textbook used.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 The diagonals of a rhombus are 24 inches and 32 inches in length. Find the number of inches in the length of a side of the rhombus. 1.....

2 A line parallel to side AB of triangle ABC intersects AC in D and CB in E. If $AD = 2$, $AC = 12$ and $BC = 18$, find the length of BE. 2.....

3 In circle O, chords AB and CD intersect at E. If $AE = 3$, $BE = 3$ and $CE = 2$, find the length of ED. 3.....

4 If the length of a diagonal of a square is 8 inches, find the number of square inches in the area of the square. 4.....

5 Two parallel lines are cut by a transversal. If two interior angles on the same side of the transversal contain x degrees and $(3x - 40)$ degrees, find the number of degrees in the smaller angle. 5.....

6 If each interior angle of an equiangular polygon contains 144 degrees, find the number of sides of the polygon. 6.....

7 Quadrilateral $ABCD$ is inscribed in a circle. If angle A contains $2x$ degrees, express the number of degrees in angle C in terms of x. 7.....

8 The coordinates of two points on a circle are $(-2,5)$ and $(4,2)$. Express in radical form the length of the chord joining the two points. 8.....

9 The coordinates of the end points of a line segment are $(-3,5)$ and $(4,7)$. Find the coordinates of the midpoint of the segment. 9.....

10 The lengths of the sides of a triangle are 8, 10 and 12. Find the perimeter of a similar triangle whose area is nine times that of the given triangle. 10.....

11 The altitude drawn to the hypotenuse of a right triangle is 8 inches long. If the length of the segments of the hypotenuse are x and $4x$, find the number of inches in the smaller segment. 11.....

12 An isosceles triangle ABC with $AC = CB$ is inscribed in circle O. At A a line is drawn tangent to circle O. If $\angle CBA$ contains 24 degrees, find the number of degrees in the *acute* angle formed by the tangent and side AC of the triangle. 12.....

13 Two concentric circles have radii of 6 inches and 10 inches, respectively. Find the number of inches in the length of a chord of the larger circle which is tangent to the smaller circle. 13.....

14 In right triangle ABC, with the right angle at C, $AB = 20$ and $\angle A = 53°$. Find to the *nearest integer* the length of AC. 14.....

[OVER]

15 Two tangents are drawn to a circle from a point outside the circle. If the angle formed by the two tangents contains 30°, find the number of degrees in the major intercepted arc. 15.....

16 The lengths of two sides of a triangle are 12 inches and 15 inches, and the included angle contains 30 degrees. Find the number of square inches in the area of the triangle. 16.....

17 If an altitude of an equilateral triangle is 12 inches, find the number of inches in the radius of the inscribed circle. 17.....

18 If the apothem of a regular hexagon is $6\sqrt{3}$, find a side of the hexagon. 18.....

19 In a circle whose radius is 20, find the length of an arc of 72 degrees. [Leave answer in terms of π.] 19.....

20 The circumference of a circle is 12π. Find the area of the circle. [Leave answer in terms of π.] 20.....

21 Write an equation of the locus of points for each of which the ordinate is 3 less than 5 times the abscissa. 21.....

Directions (22–26): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement or answers the question.

22 A student has proved the theorem: The diagonals of a rectangle are equal. He then reasons as follows, "In quadrilateral $ABCD$, diagonal AC = diagonal BD. Therefore, $ABCD$ is a rectangle." His reasoning is

(1) unsound, and an example of circular reasoning
(2) unsound, and an example of reasoning from a converse
(3) sound, and an example of reasoning from an inverse
(4) sound, and an example of indirect reasoning 22.....

23 In isosceles triangle ABC, $AC = CB$. If D is a point on the base AB lying between A and B, and CD is drawn, then

(1) $AC > CD$ (3) $\angle A > \angle ADC$
(2) $CD > AC$ (4) $\angle B > \angle BDC$ 23.....

24 If in quadrilateral $ABCD$, $AB = CD$ and $AD = BC$, it must necessarily follow that the diagonals AC and BD

(1) are equal (3) bisect each other
(2) are perpendicular (4) bisect the angles of the quadrilateral 24.....

25 A triangle is inscribed in a circle. If the length of the radius of the circle is equal to one-half the length of a side of the triangle, then the triangle must be

(1) acute (3) right
(2) obtuse (4) equilateral 25.....

26 The perimeter of an equilateral triangle is 12. The area is

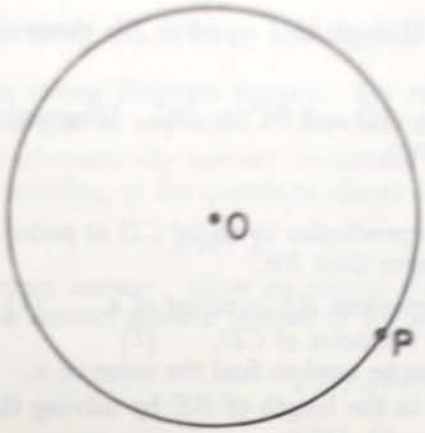
(1) $2\sqrt{3}$ (3) $6\sqrt{3}$
(2) $4\sqrt{3}$ (4) $36\sqrt{3}$ 26.....

27 If two circles are externally tangent to each other, what is the greatest number of common tangents that can be drawn to both circles? 27.....

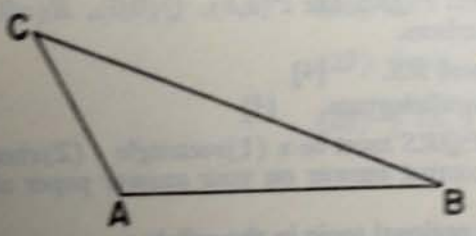
28 Write an equation of the straight line which passes through the point (1,4) and is parallel to the x-axis. 28.....

Directions (29–30) : Leave all construction lines on the paper.

29 Construct a line tangent to circle O at point P.



30 Locate by construction the *center* of the circle circumscribed about the triangle ABC. Label the center O.



[3]

[OVER]

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove *either a* or *b* but *not* both: [10]

a If two angles of a triangle are equal, the sides opposite these angles are equal.

OR

b If the three angles of one triangle are equal to the three angles of another triangle, the triangles are similar.

32 In quadrilateral $ABCD$, $AB = AD$ and $BC = DC$. Diagonal AC is extended through C to E and lines BE and DE are drawn.
Prove: $BE = DE$ [10]

33 In circle O, diameter AB is perpendicular to chord CD at point E. The length of chord CD is 12 inches, and AE is 5 inches longer than BE.

a If BE is represented by x, represent AE in terms of x. [1]

b State a reason why E is the midpoint of CD. [2]

c Write an equation which can be used to find the value of x. [3]

d Find the number of inches in the length of BE by solving the equation obtained in answer to part *c*. [3]

e Find the number of inches in the length of the diameter AB. [1]

34 Given: A fixed point P is 3 inches from line l.

a Describe fully the locus of points which are 3 inches from P. [3]

b Describe fully the locus of points which are d inches from l. [3]

c How many points are there which satisfy the conditions given in both *a* and *b* if

(1) $d = 3$? [2]

(2) $d = 6$? [2]

35 The vertices of triangle ABC are A (—4,—2), B (4,4) and C (—6,9).

a Using graph paper, plot these vertices and draw the triangle. [1]

b Show that triangle ABC is isosceles. [4]

c Find the length of the altitude of the triangle drawn to side AB. [3]

d Find the area of triangle ABC. [2]

36 The diagonals of a rectangle are each 20 inches long and intersect at an angle of 108°. Find to the *nearest tenth* the number of inches in the length and the width of the rectangle. [6, 4]

*37 The vertices of quadrilateral $PQRS$ are $P(a,b)$, $Q(0,0)$, $R(c,0)$ and $S(a + c,b)$, in which a, b and c are positive numbers.

a Find the slopes of PQ and RS. [4]

b Show that $PQRS$ is a parallelogram. [4]

c If $c = \sqrt{a^2 + b^2}$, then $PQRS$ must be a (1) rectangle (2) rhombus (3) square [Write the *number* preceding the correct answer on your answer paper after the letter *c*.] [2]

* This question is based on an optional topic in the syllabus.

FOR TEACHERS ONLY

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SCORING KEY
TENTH YEAR MATHEMATICS

Monday, January 27, 1964 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 22–26, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) 20

(2) 3

(3) $4\frac{1}{2}$

(4) 32

(5) 55

(6) 10

(7) $180-2x$

(8) $\sqrt{45}$ *or* $3\sqrt{5}$

(9) $(\frac{1}{2}, 6)$

(10) 90

(11) 4

(12) 24

(13) 16

(14) 12

(15) 210

(16) 45

(17) 4

(18) 12

(19) 8π

(20) 36π

(21) $y = 5x - 3$

(22) 2

(23) 1

(24) 3

(25) 3

(26) 2

(27) three

(28) $y = 4$

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(33) *a* $x + 5$ [1]
b A diameter perpendicular to a chord of a circle bisects the chord. [2]
c $x^2 + 5x = 36$ [3]
d 4 [3]
e 13 [1]

(34) *a* A circle with center at P and a radius of 3 inches [3]
b Two lines, one on each side of l, d inches from l and parallel to l [3]
c (1) 2 [2]
(2) 1 [2]

(35) *c* 10 [3]
d 50 [2]

(36) Length = 16.2 [6]
Width = 11.8 [4]

*(37) *a* $\frac{b}{a}$, $\frac{b}{a}$ [4]
c 2 [2]