The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Thursday, January 24, 1963 — 1:15 to 4:15 p.m., only

Name of pupil..... Name of school.....

Name and author of textbook used.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 In $\triangle ABC$ a line parallel to AC intersects AB at D and BC at E. If $BD = 6$, $DA = 2$ and $DE = 3$, find the length of AC. 1.....

2 Find the area of a square whose diagonal is 12. 2.....

3 Quadrilateral $ABCD$ is inscribed in a circle. If $\overset{\frown}{AB} = 119°$, $\overset{\frown}{BC} = 73°$ and $\overset{\frown}{CD} = 60°$, what is the number of degrees in angle ABC? 3.....

4 Two chords of a circle, AB and CD, intersect at E so that $CE = 6$ and $ED = 8$. If AE is represented by x and EB by $3x$, find the value of x. 4.....

5 One angle of a rhombus is 60° and a side is 8. Find the area of the rhombus. 5.....

6 If the bases of an isosceles trapezoid are 15 feet and 21 feet, respectively, and each leg is 5 feet, find the number of square feet in the area of the trapezoid. 6.....

7 What are the coordinates of the midpoint of the line segment whose end points are $(-2,5)$ and $(3,-9)$? 7.....

8 An interior angle of a regular polygon contains 160°. Find the number of sides of the polygon. 8.....

9 In a circle whose radius is 6 inches, find the number of degrees in the central angle of an arc whose length is 4π inches. 9.....

10 The areas of two similar polygons are in the ratio 1:25. If a side of the smaller polygon is 3, find the length of the corresponding side of the larger polygon. 10.....

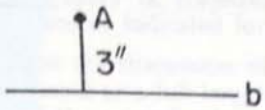
11 Find the length of the line segment joining the points whose coordinates are $(2,-4)$ and $(8,4)$. 11.....

12 The area of a regular polygon is 81 and its perimeter is 36. Find the length of its apothem. 12.....

13 In right $\triangle ABC$, CD is the altitude drawn to the hypotenuse AB. If $CD = 2$ and $AD = 1$, find BD. 13.....

14 From a point outside a circle a tangent and a secant are drawn to the circle. The circle divides the secant into an external segment of 4 inches and an internal segment of 21 inches. Find the number of inches in the length of the tangent. 14.....

15 Point A is 3 inches from line b as shown in the diagram. In the plane that contains point A and line b, what is the total number of points which are 6 inches from A and also 1 inch from b?



15.....

16 Corresponding altitudes of two similar triangles are 6 and 4. If the perimeter of the larger triangle is 18, what is the perimeter of the smaller triangle? 16.....

17 In circle O with radius 17 chord CD is 30. Find the distance of chord CD from the center of the circle. 17.....

18 The angle formed by two tangents to a circle from the same point is 70°. Find the number of degrees in the smaller intercepted arc. 18.....

19 In triangle ABC, angle $A = 58°$ and $AB = 14$. Find to the *nearest integer* the length of the altitude to side AC. 19.....

20 The legs of a right triangle are 3 inches and 4 inches. Find the number of inches in the length of the median to the hypotenuse. 20.....

21 Triangle ABC is isosceles with $CA = CB$. If side CA is extended through A to P and PB is drawn, which is the longest side of triangle PAB? 21.....

22 In trapezoid $ABCD$ with bases AB and DC, diagonals AC and BD intersect in point E. Name a triangle which is equal in area to $\triangle ABD$. 22.....

Directions (23–29): Indicate the correct completion for *each* of the following by writing on the line at the right the number 1, 2, 3 or 4.

23 A parallelogram must be a rectangle if its diagonals
(1) bisect each other
(2) bisect the angles
(3) are perpendicular to each other
(4) are equal
23.....

24 A proof which lists all possible conclusions and shows that all but one of these conclusions lead to contradictions is classified as
(1) proof by counterexample
(2) indirect proof
(3) overdetermined proof
(4) circular proof
24.....

25 The locus of points in the coordinate plane at a distance of 5 units from the x-axis consists of the graph(s) of the equation(s)
(1) $y = 5$
(2) $x = 5$
(3) $y = 5$, $y = -5$
(4) $x = 5$, $x = -5$
25.....

26 If an altitude of an equilateral triangle is $5\sqrt{3}$, the length of a side is
(1) 10
(2) 5
(3) $10\sqrt{3}$
(4) $5\sqrt{3}$
26.....

27 The coordinates of the vertices of quadrilateral $ABCD$ are A (0,0), B (9,0), C (10,3) and D (1,3), respectively. The area of $ABCD$ is

(1) $9\sqrt{10}$
(2) $\frac{9}{2}\sqrt{10}$
(3) $\frac{27}{2}$
(4) 27

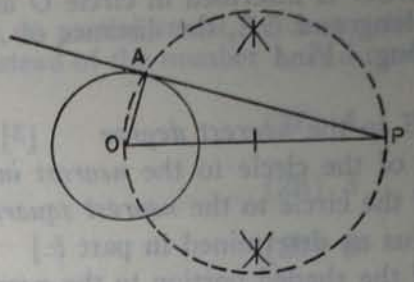
27.....

28 Side AC of triangle ABC is extended through C to D. Angle $BCD = 108°$ and the number of degrees in angle A is twice the number of degrees in angle B. Triangle ABC is

(1) right
(2) obtuse
(3) isosceles
(4) scalene

28.....

29 The accompanying diagram shows the construction of a tangent PA to circle O from external point P. Which proposition is used in the proof of this construction to show that PA is a tangent?

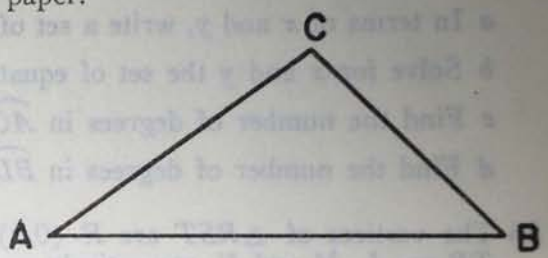


(1) Tangents to a circle from an external point are equal.
(2) If two circles intersect, the line joining their centers is the perpendicular bisector of the common chord.
(3) An angle inscribed in a semicircle is a right angle.
(4) If a tangent and a secant are drawn to a circle from the same point, the tangent is the mean proportional between the secant and its external segment.

29.....

Directions (30): Leave all construction lines on the paper.

30 On line segment AB locate, by construction, a point M that is equidistant from CA and CB.



Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove *either* *a* or *b*: [10]

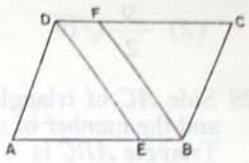
a If two sides of a triangle are equal, the angles opposite these sides are equal.

OR

b The area of a trapezoid is equal to one-half the product of the altitude and the sum of the bases.

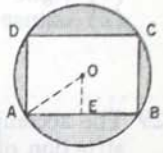
32 In $\triangle ABC$ altitudes AD and CE intersect in point H.
Prove: $\frac{BD}{HD} = \frac{AD}{DC}$ [10]

33 In the accompanying figure $ABCD$ is a parallelogram. The bisector of angle B meets DC in F and the bisector of angle D meets AB in E.



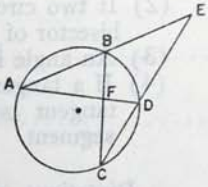
Prove:
a $AE = FC$ [7]
b $EB = DF$ [3]

34 Rectangle $ABCD$ is inscribed in circle O as shown in the figure. Side AB is 12 inches long and OE, the distance of AB from the center of the circle, is 4 inches long. Find



a angle AOE to the *nearest degree* [3]
b the radius of the circle to the *nearest inch* [3]
c the area of the circle to the *nearest square inch* [Use the approximation $\pi = \frac{22}{7}$ and the length of the radius as determined in part *b*.] [2]
d the area of the shaded portion to the *nearest square inch* [2]

35 In the accompanying figure chords AB and CD are extended to meet at E. Chords AD and BC intersect at F. Angle $AEC = 51°$ and angle $AFC = 92°$. Let the number of degrees in $\overset{\frown}{AC}$ be represented by $(23y - 8x)$ and the number of degrees in $\overset{\frown}{BD}$ by $(y + 4x)$.



a In terms of x and y, write a set of equations that can be used to solve for x and y. [2, 2]
b Solve for x and y the set of equations written in answer to part *a*. [4]
c Find the number of degrees in $\overset{\frown}{AC}$. [1]
d Find the number of degrees in $\overset{\frown}{BD}$. [1]

36 The vertices of $\triangle RST$ are R (0,0), S (2*a*,2*b*) and T (4*a*,0). The midpoints of RS, ST and TR are L, M and N, respectively.
a Express the coordinates of L, M and N in terms of a and b. [3]
b Express the lengths of the medians from R, S and T in terms of a and b. [6]
c $\triangle RST$ must be (1) equilateral (2) right (3) isosceles (4) scalene [Write the *number* preceding the correct answer on your answer paper after the letter *c*.] [1]

*37 *a* Given acute $\triangle ABC$ with sides a, b and c opposite angles A, B and C, respectively. Starting with the formula $K = \frac{1}{2}bh$ for the area of $\triangle ABC$, show that $K = \frac{1}{2}ab \sin C$ is also a formula for the area of $\triangle ABC$. [5]
b Using the formula $K = \frac{1}{2}ab \sin C$, find the number of degrees in angle C if angle C is acute, $a = 20$, $b = 5$ and $K = 25$. [5]

*This question is based on an optional topic in the syllabus.

FOR TEACHERS ONLY

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SCORING KEY

TENTH YEAR MATHEMATICS

Thursday, January 24, 1963 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 23–29, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) 4

(2) 72

(3) 84

(4) 4

(5) $32\sqrt{3}$ *or* 55.4

(6) 72

(7) $(\frac{1}{2}, -2)$

(8) 18

(9) 120

(10) 15

(11) 10

(12) $4\frac{1}{2}$

(13) 4

(14) 10

(15) 4

(16) 12

(17) 8

(18) 110

(19) 12

(20) $2\frac{1}{2}$

(21) *BP*

(22) *ABC*

(23) 4

(24) 2

(25) 3

(26) 1

(27) 4

(28) 3

(29) 3

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(34) *a* 56 [3]
b 7 [3]
c 154 [2]
d 58 [2]

(35) *a* $\frac{1}{2}(22y - 12x) = 51$ [2]
$\frac{1}{2}(24y - 4x) = 92$ [2]

OR

$11y - 6x = 51$
$6y - x = 46$

b $x = 8, y = 9$ [4]
c 143 [1]
d 41 [1]

(36) *a* $L\ (a,b),\ M\ (3a,b),\ N\ (2a,0)$ [3]
b $RM = \sqrt{9a^2 + b^2}$
$SN = 2b$
$TL = \sqrt{9a^2 + b^2}$ [6]
c 3 [1]

(37) *b* 30 [5]