The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Thursday, January 25, 1962 — 1:15 to 4:15 p.m., only

Name of pupil..... Name of school.....

Name and author of textbook used.....

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 The perimeter of an equilateral triangle is 12. Find its area. 1.....

2 Chords AB and CD intersect inside circle O at point E. If $AE = 5\frac{1}{7}$, $EB = 2\frac{1}{3}$ and $DE = 4$, find EC. 2.....

3 The areas of two similar triangles are in the ratio 4:9. The shortest side of the first triangle is 4. Find the shortest side of the second triangle. 3.....

4 The area of a rhombus is 49 and the length of one diagonal is twice that of the other. Find the length of the shorter diagonal. 4.....

5 The radius of a circle is 6. Find the length of a diagonal of an inscribed rectangle. 5.....

6 Quadrilateral $ABCD$ is inscribed in a circle. The number of degrees in angle A is represented by $90 - x$. Express in terms of x the number of degrees in angle C. 6.....

Directions (7–14): Write on the line at the right of *each* of the following the *number* preceding the expression that best completes the statement or answers the question.

7 If two regular polygons have the same number of sides, they must be

(1) equal in area
(2) similar
(3) equal in perimeter
(4) inscribed in the same circle

7.....

8 Two circles with radii 4 and 8, respectively, are internally tangent to each other. The distance between their centers is

(1) 0
(2) 2
(3) 12
(4) 4

8.....

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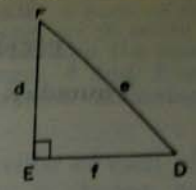
9 In right triangle DEF, shown in the accompanying diagram, the length of the hypotenuse can be expressed as



(1) $\frac{d}{\sin D}$ (3) $\frac{f}{\sin D}$

(2) $d \sin D$ (4) $f \cos D$ 9.....

10 The difference between the supplement and the complement of an acute angle is

(1) an acute angle (3) an obtuse angle

(2) a right angle (4) a straight angle 10.....

11 Which definition may be criticized because one part of it may be deduced from the remaining part?

(1) A regular hexagon is a hexagon that is both equilateral and equiangular.

(2) A trapezoid is a quadrilateral having one pair of sides parallel and one pair of opposite sides not parallel.

(3) A rectangle is a parallelogram one of whose angles is a right angle.

(4) A parallelogram is a quadrilateral having both pairs of opposite sides parallel and equal. 11.....

12 The coordinates of vertices A and B of an equilateral triangle ABC are $(-4, 0)$ and $(4, 0)$, respectively. The coordinates of C may be

(1) $(0, 2\sqrt{3})$ (3) $(4, 4\sqrt{3})$

(2) $(0, 4\sqrt{3})$ (4) $(0, 4)$ 12.....

13 An example of circular reasoning always occurs in

(1) proving two chords equal by showing that their arcs are equal

(2) using in the same proof the postulates, "Halves of equals are equal" and "Doubles of equals are equal"

(3) using a theorem as an authority in the proof of that theorem

(4) using a theorem as an authority in the proof of the converse of that theorem 13.....

14 Which point lies farthest from the origin?

(1) $(0, -9)$ (3) $(8, 5)$

(2) $(-7, 6)$ (4) $(-2, 9)$ 14.....

15 The altitude to the hypotenuse of a right triangle divides the hypotenuse into segments of 3 and 9. Find the shorter leg of the triangle. 15.....

16 Find the number of degrees in the smallest *obtuse* angle that can be an interior angle of a regular polygon. 16.....

17 The angles of a quadrilateral are in the ratio $3:4:5:6$. Find the number of degrees in the smallest angle of the quadrilateral. 17.....

18 The coordinates of A and B are, respectively, $(c, 3d)$ and $(5c, -3d)$. Find the coordinates of the midpoint of the line segment AB. 18.....

19 The bases of an isosceles trapezoid are 7 and 17 and the legs are each 13. Find the distance between the bases. 19.....

Directions (20–24) : If the blank space in each statement below is replaced by the word *always*, *sometimes* (but not always) or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

20 The area of a triangle is ... equal to one-half the product of two of its sides. 20.....

21 If the opposite sides of a polygon are parallel, the polygon will ... be a parallelogram. 21.....

22 In a circle, if two chords are at unequal distances from the center, the greater chord ... lies nearer to the center. 22.....

23 The three medians of a triangle ... intersect outside the triangle. 23.....

24 If the lengths of the three altitudes and the lengths of the three sides of an acute scalene triangle are listed in decreasing order of size, the last length in the list will ... be that of an altitude. 24.....

25 The area of a circle is $\frac{9\pi}{4}$. Find its circumference. 25.....

26 The radius of a circle is 8. Find the number of degrees in the central angle of a sector having an area of 16π. 26.....

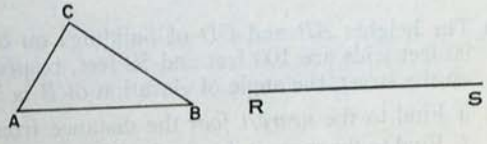
27 Given line m in a plane. What is the number of points in that plane at a fixed distance d (d not equal to zero) from m and also equidistant from two points, A and B, on m? 27.....

28 Side AB of a regular polygon inscribed in circle O makes an angle of $20°$ with the tangent to circle O at A. Find the number of sides of the polygon. 28.....

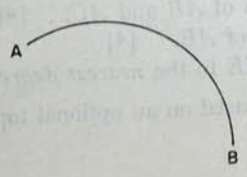
Directions (29–30) : Leave all construction lines on the paper.

29 Given triangle ABC, construct triangle RST on base RS so that

$$\frac{AB}{RS} = \frac{AC}{RT} = \frac{BC}{ST}.$$



30 Find by construction the center O of the circle of which AB is an arc.



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Part II

Answer four questions from this part. Show all work unless otherwise directed.

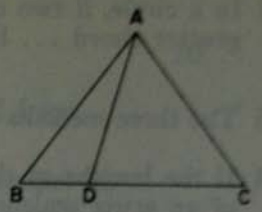
31 Prove *either a* or *b*: [10]

a A diameter perpendicular to a chord of a circle bisects the chord and its arcs.

OR

b If three angles of one triangle are equal to three angles of another triangle, the triangles are similar.

32 In triangle ABC, $AB = AC$ and D is a point on base BC. Prove that $AC > AD$. [10]



33 Prove that if two altitudes of a triangle intersect inside the triangle, the product of the segments of one altitude equals the product of the segments of the other. [10]

34 Any parallelogram $ABCD$ may be placed on the XY-plane so that the coordinates of its vertices are $A(0, 0)$, $B(a, 0)$, $C(a + b, c)$ and $D(b, c)$. Use the fact stated above to verify by the methods of coordinate geometry that

a the diagonals of a parallelogram bisect each other [4]

b the opposite sides of a parallelogram are equal [6]

35 Diameter AB of circle O is 12 inches in length. AB is extended through B to a point C and tangent CD is drawn, meeting the circle at D. CD is 2 inches longer than BC.

a Let x represent the number of inches in BC, and express CD and CA in terms of x. [2]

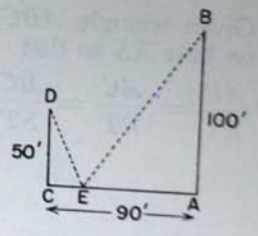
b Express as an equation in the variable x, the relationship that exists among BC, CA and CD. [3]

c Find the length of BC by solving the equation in part *b*. [5]

36 The heights AB and CD of buildings on opposite sides of a street 90 feet wide are 100 feet and 50 feet, respectively. From a point E on the street, the angle of elevation of B is 55°.

a Find to the *nearest foot* the distance from E to A. [4]

b Find to the *nearest degree* the angle of elevation of D from E. [6]



*37 The coordinates of the vertices of triangle ABC are $A(2, 3)$, $B(9, -2)$ and $C(5, 8)$. Median AE is drawn.

a Find the slopes of AB and AC. [4]

b Find the slope of AE. [4]

c Find angle CAE to the *nearest degree*. [2]

* This question is based on an optional topic in the syllabus.

FOR TEACHERS ONLY

10 INSTRUCTIONS FOR RATING TENTH YEAR MATHEMATICS

Thursday, January 25, 1962 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 7–14, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $4\sqrt{3}$ *or* 6.9
(2) 3
(3) 6
(4) 7
(5) 12
(6) $90 + x$
(7) 2
(8) 4
(9) 1
(10) 2
(11) 4
(12) 2
(13) 3
(14) 3
(15) 6
(16) 108
(17) 60
(18) $(3c, 0)$
(19) 12
(20) sometimes
(21) sometimes
(22) always
(23) never
(24) always
(25) 3π *or* 9.4
(26) 90
(27) 2
(28) 9

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely a mechanical one or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent, while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent, depending on the relative importance of the principle in the solution of the problem.

(35) *a* $CD = x + 2$, $CA = 12 + x$ [2]

b $\frac{x}{x+2} = \frac{x+2}{x+12}$ *or* $x^2 + 12x = x^2 + 4x + 4$ [3]

c $BC = \frac{1}{2}$ inch [5]

(36) *a* 70 [4]
b 68 [6]

*(37) *a* Slope of $AB = -\frac{5}{7}$, slope of $AC = \frac{5}{3}$ [4]

b Slope of $AE = 0$ [4]

c 59 [2]