TENTH YEAR MATHEMATICS—JANUARY 1957 (I)

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1. Find a diagonal of a rectangle whose sides are 3 and 7.

2. The angles of a triangle are in the ratio 1:3:5. Find the number of degrees in the *smallest* angle of the triangle.

3. The area of a rhombus is 48 and one diagonal is 12. Find the other diagonal.

4. In parallelogram $ABCD$, $AB = 12$, $AD = 4$ and angle $A = 30°$. Find the area of the parallelogram.

5. Find the number of degrees in the sum of the interior angles of a polygon of ten sides.

6. A side of a regular polygon is s and the apothem is a. If the area of the polygon equals $8as$, find the number of sides in the polygon.

7. In triangle ABC, angle $C = 118°$ and angle $B = 44°$. Which is the *shortest* side of the triangle?

8. In triangle ABC, angle C is a right angle, $AC = 8$ and $BC = 20$. Find angle B to the *nearest degree*.

9. In circle O, central angle $AOC = 140°$. If B is a point on major arc AC, find the number of degrees in angle ABC.

10. In a circle, chords AB and CD intersect at E. If $AE = 9$, $EB = 3$ and $CE = 6$, find ED.

11. In a circle, chord $AB = 16$ and its distance from the center is 6. Find the radius of the circle.

12. Find the area of a circle whose radius is 5.

13. $ABCD$ is a square inscribed in a circle. A tangent is drawn at A. Find the number of degrees in the *acute* angle between the tangent and side AB.

14. Given points $A(-4, -3)$ and $B(6, -2)$. Find the coordinates of the midpoint of AB.

15. Find the length of the line segment joining the points $(-1, 4)$ and $(7, 1)$.

16. Write the equation of the locus of points whose ordinate is 2 more than one-half its abscissa.

17. The altitude of an equilateral triangle is 12. Find the radius of the inscribed circle.

18. All members of the math team are honor students. Which of the following statements expresses a conclusion that logically follows from the given statement? [Indicate your answer by writing the letter *a*, *b* or *c* on the line at the right.]

a. If John is an honor student, he is on the math team.
b. If Mary is not on the math team, she is not an honor student.
c. If Fred is not an honor student, he is not on the math team.

Directions (19-24): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b*, *c* or *d*.

19. The sides of an isosceles triangle are *a*, *a* and *b*. The perimeter of the triangle formed by joining the midpoints of the sides of the given triangle is (*a*) $\frac{1}{2}(a+b)$ (*b*) $a+\frac{1}{2}b$ (*c*) $\frac{3}{2}a+b$ (*d*) $2a+b$

20. The number of points equidistant from two intersecting lines and at a distance *d* from their point of intersection is (*a*) 1 (*b*) 2 (*c*) 8 (*d*) 4

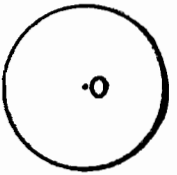
21. The diagonals of an isosceles trapezoid (*a*) are perpendicular to each other (*b*) are equal (*c*) bisect each other (*d*) bisect the angles of the trapezoid

22. In triangle *ABC*, if *AB* is greater than *BC* and *BC* is greater than *AC*, then (*a*) angle *C* is the largest angle (*b*) angle *A* is the largest angle (*c*) angle *C* is the smallest angle (*d*) angle *A* is the smallest angle

23. The median to the hypotenuse of a right triangle divides the triangle into two triangles which are (*a*) scalene triangles equal in area (*b*) scalene triangles unequal in area (*c*) isosceles triangles equal in area (*d*) isosceles triangles unequal in area

24. Similar triangles are correctly defined as triangles (*a*) whose corresponding sides are proportional (*b*) whose corresponding angles are equal (*c*) whose corresponding angles are equal and whose corresponding sides are proportional (*d*) which have two angles of one triangle equal to two angles of the other

25. Inscribe a regular hexagon in circle *O*.



Part II

Answer three questions from this part.

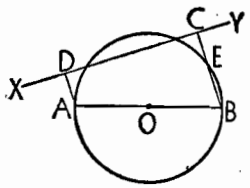
26. Prove: The sum of the angles of a triangle is equal to a straight angle. [10]

27. In triangle ABC, BD bisects angle B and intersects AC at D. Through C a line parallel to BD is drawn to meet AB extended in E. Prove that

a. triangle EBC is isosceles [6]

b. $AD \times BC = AB \times CD$ [4]

28. In the diagram at the right, AB is a diameter of circle O. AD and BC are perpendicular to secant XY. CB intersects the circle at E. Prove: $CE = AD$. [Suggestion: Draw AE.] [10]



29. Prove: The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides. [10]

30. $ABCD$ is a parallelogram. E is the midpoint of diagonal BD. Through E a line is drawn intersecting BC in F and AD in G. Prove that

a. triangle DEG is congruent to triangle FEB [5]

b. the area of quadrilateral $ABEG$ is equal to the area of quadrilateral $CDEF$ [5]

*31. *a.* On a set of coordinate axes, plot the points $A(1, 1)$, $B(10, 4)$, $C(7, 7)$, $D(7, 3)$ and $E(5, 5)$. [2]

b. Using the formula for the slope of a line, show that A, E and C lie on the same straight line. [3]

c. Point D lies on AB. Show that triangle ADE is similar to triangle ABC. [5]

* This question is based on one of the optional topics in the syllabus and may be substituted for any question in *either* part II or part III.

Part III

Answer two questions from this part. Show all work.

32. The longer base of an isosceles trapezoid exceeds the shorter base by 6, and each leg equals 5.

a. If the shorter base is represented by x, express the longer base in terms of x. [1]

b. Find the altitude of the trapezoid. [3]

c. If the area of this trapezoid equals 28, find each base. [5, 1]

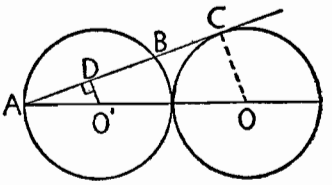
33. Two sides of a triangular plot of ground meet at an angle of 76°. One of these sides is 60 feet and the other is 140 feet.

a. Find to the *nearest tenth of a foot* the altitude to the side whose length is 140 feet. [3]

b. Find the area of the triangle to the *nearest square foot.* [4]

c. If a drawing of this plot is made on a scale of 1 inch = 20 feet, find the area of this drawing to the *nearest square inch.* [3]

34. Equal circles O and O' are tangent to each other. OO' is extended to meet circle O' at A. AC is tangent to circle O at C and intersects circle O' at B. $O'D$ is perpendicular to AC. The radius of each circle is 6. Find AO, $O'D$, AD, AC, BC. [Answers may be left in radical form.] [2, 2, 2, 2, 2]



35. Plot the points $A(2, 5)$, $B(11, 2)$, $C(9, 8)$ and $D(4, 8)$. Draw AB, BC, CD and DA. Find the area of $ABCD$. [2, 8]