APPENDIX B

SPECIMEN REGENTS EXAMINATIONS

THE UNIVERSITY OF THE STATE OF NEW YORK

TENTH-YEAR MATHEMATICS

January, 1955

PART I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed.

Directions (questions 1–4): Write on the line at the right of *each* question the expression that, when inserted in the blank, will make the statement true.

1. If two parallel lines are cut by a transversal, the two interior angles on the same side of the transversal are . . . .

2. If line a is perpendicular to line b and line b is parallel to line c, then a is . . . to c.

3. The angle formed by two secants intersecting outside the circle is measured by one half the . . . of the intercepted arcs.

4. The areas of two parallelograms which have equal bases are to each other as their . . . .

Directions (5–13): Write on the line at the right the correct answer to *each* numerical exercise.

5. The line segment joining the midpoints of two sides of an equilateral triangle is 5. Find the *perimeter* of the triangle.

6. Two polygons are similar and the area of one is 9 times the area of the other. If a side of the smaller polygon is 5, find the corresponding side of the larger polygon.

7. AB is the hypotenuse of right triangle ABC and CD is the altitude on the hypotenuse. If $AD = 2$ and $CD = 8$, find DB.

8. Two chords, AB and CD, of a circle intersect at P. $AP = 1$ and $PB = 26$. If CP is represented by x and PD by $2x$, find x.

9. Find the length of an arc whose central angle is 10° in a circle whose radius is 9 inches. [Answer may be left in terms of π.]

10. Find the area of a circle whose radius is 7. [Answer may be left in terms of π.]

11. Find the coordinates of the midpoint of the line segment whose end points are $A\ (-2, 5)$ and $B\ (4, 9)$.

12. Find the distance from point $A\ (3, 4)$ to point $B\ (8, 7)$. [Answer may be left in radical form.]

13. Write an equation of the straight line which is the locus of points, each of which has an abscissa equal to 5.

Directions (questions 14–17): For *each* of the following, if the statement is *always true*, write *true* on the line at the right; if it is *not always* true, or *never* true, write the word *false*.

14. If the diagonals of a quadrilateral are equal, the quadrilateral is a rectangle.

15. Two right triangles are similar if the legs of one are proportional to the legs of the other.

16. A diameter which bisects one of two parallel chords, neither of which is a diameter, bisects the other also.

17. If two circles are externally tangent to each other, the greatest number of common tangents that can be drawn to both circles is four.

Directions (questions 18–22): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b* or *c*.

18. The center of a circle inscribed in a triangle is the point of intersection of (*a*) the bisectors of its angles (*b*) its altitudes (*c*) the perpendicular bisectors of its sides

19. If each interior angle of a regular polygon is 140°, the number of sides of the polygon is (*a*) seven (*b*) eight (*c*) nine

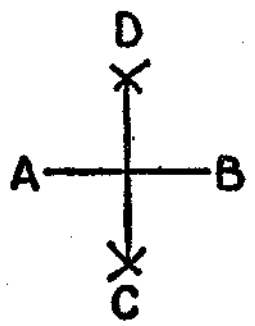
20. A median of a triangle divides the triangle into two triangles which are always (*a*) similar (*b*) equal in area (*c*) congruent

21. If, in triangle ABC, angle $A = 50°$ and angle $B = 64°$, the longest side of the triangle is (*a*) AC (*b*) BC (*c*) AB

22. The area of an equilateral triangle whose side is 8 is (*a*) $16\sqrt{3}$ (*b*) $4\sqrt{3}$ (*c*) $16\sqrt{2}$

23. Since it has been proved that the diagonals of a parallelogram bisect each other and since a trapezoid is not a parallelogram, it follows that the diagonals of a trapezoid do not bisect each other. Is the above statement an illustration of sound reasoning? [Answer *yes* or *no*.]

24. The accompanying figure shows the usual method of constructing the perpendicular bisector of a given line segment. Which statement, *a* or *b*, may be used to prove that CD is the perpendicular bisector of AB?



a. All points on the perpendicular bisector of a line segment are equidistant from its end-points.

b. Two points each equidistant from the end-points of a line segment determine its perpendicular bisector.

Directions (25): Leave all construction lines on the paper.

25. Given point P on line m. Construct the locus of the centers of circles tangent to line m at point P.

PART II

Answer three questions from this part.

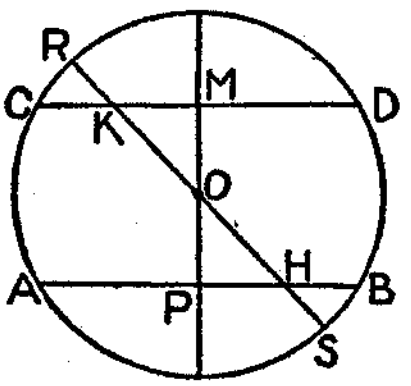
26. *a.* *Prove:* An angle inscribed in a circle is measured by one half its intercepted arc. [Prove only the case in which one side of the angle is a diameter of the circle.] [8]

b. Tell which of the following axioms is used to prove this theorem for the case in which the center of the circle lies outside the angle: [2]

(1) If equals are added to equals, the sums are equal.

(2) If equals are subtracted from equals, the remainders are equal.

27. Two chords AB and CD of a circle whose center is O are equal and parallel. A line through O perpendicular to AB intersects AB at P and CD at M. Diameter RS of the circle intersects AB at H and CD at K. Prove that:

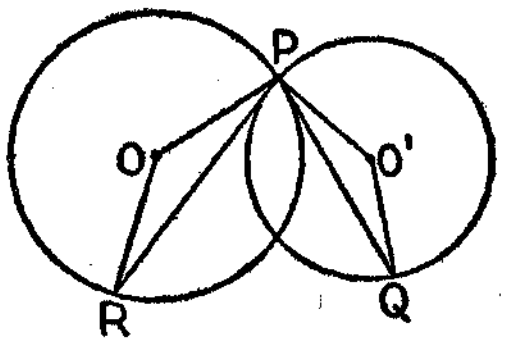


a. triangles HOP and KOM are congruent [6]

b. $HS = KR$ [4]

28. Prove: The square of the hypotenuse of a right triangle is equal to the sum of the squares of the legs. [10]

29. In the accompanying figure unequal circles O and O' intersect at P. Chord PR is tangent to circle O' and chord PQ is tangent to circle O. Radii OP, OR, $O'P$, and $O'Q$ are drawn.



a. Prove that
(1) angle OPR = angle $O'PQ$ [5]
(2) triangles OPR and $O'PQ$ are similar [3]

b. If the ratio of chord PR to chord PQ is $a:b$, find the ratio of the area of circle O to the area of circle O'. [2]

30. Perpendicular lines r and s intersect at O, and P is a point on r.

a. State *in full* the locus of points which are
(1) at a given distance k from P [3]
(2) equidistant from r and s [3]

b. Find the number of points satisfying *both* conditions given in *a* if
(1) $OP = 6$ and $k = 8$ [1]
(2) $OP = 6$ and $k = 3\sqrt{2}$ [2]
(3) $OP = 6$ and $k = 3$ [1]

*31. *a.* Using graph paper, on the same set of axes draw the graph of *each* of the following equations: (1) $y = 6$, (2) $y = 2x$, and (3) $y = x + 6$. [2, 2, 3]

b. Find the coordinates of the vertices of the triangle whose sides are the line segments joining the points of intersection of the graphs made in answer to *a*. [3]

* This question is based on one of the optional topics in the syllabus and may be substituted for any question in *either* part II or part III.

PART III

Answer two questions from this part. Show all work.

32. The base of a triangle is represented by x and its altitude by $x + 2$. The bases of a trapezoid are 11 and 4 and its altitude is represented by $x - 2$.

a. Express the area of the triangle in terms of x. [2]

b. Express the area of the trapezoid in terms of x. [2]

c. If the triangle and the trapezoid are equal in area, find x. [4]

d. Using the larger value of x found in answer to *c*, find the area of the triangle. [1]

e. If the smaller value of x found in answer to *c* is used, will the triangle be equal in area to the trapezoid? [1]

33. The vertices of triangle ABC are A (1, 2), B (8, 3), and C (4, 6).

a. Show that the triangle is isosceles. [5]

b. Show that angle C is a right angle. [5]

34. In rhombus $ABCD$ diagonal AC is drawn. If $AC = 20$ and angle $BAC = 27°$, find

a. diagonal BD to the *nearest tenth* [4]

b. the area of the rhombus to the *nearest integer* [2]

c. side AB to the *nearest integer* [4]

35. AB is a diameter of a circle whose center is O. OC is a radius perpendicular to AB. K is a point on radius OB. Line CK is drawn and extended to meet the circle at D. At D a tangent is drawn to the circle and meets AB extended at P.

a. Draw and letter a figure which represents the conditions given above. [2]

b. If the number of degrees in arc DB is represented by $2x$, show that angle CDP and angle DKB are equal. [4]

c. If $AP = 18$ and $KP = 12$, find the radius of the circle. [4]

June, 1956

PART I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1. The hypotenuse of a right triangle is 7 and one leg is 4. Find the other leg.

2. The angles of a triangle are in the ratio 3:5:7. Find the number of degrees in the *smallest* angle of the triangle.

3. Two angles are complementary and one angle is 48° greater than the other. Find the number of degrees in the *smaller* angle.

4. Find the number of degrees in the sum of the interior angles of a polygon of twelve sides.

5. The bases of an isosceles trapezoid are 9 and 15, and each base angle contains 45°. Find the altitude of the trapezoid.

6. Find a side of an equilateral triangle whose area is $16\sqrt{3}$.

7. Find the area of a triangle whose base is 12 and whose altitude is 7.

8. Find the area of a circle whose radius is 3.

9. Find the length of an arc of 70° in a circle whose radius is 9.