TENTH YEAR MATHEMATICS

Monday, January 25, 1954 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in tenth year mathematics, (*c*) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year.

Part II

Answer three questions from part II.

26 Prove: Two right triangles are congruent if the hypotenuse and a leg of one are equal to the hypotenuse and a leg of the other.   [10]

27 Diagonal AC of parallelogram $ABCD$ is extended through A to point E and through C to point F, making CF equal to AE. Lines ED, DF, FB and BE are drawn. Prove that $EDFB$ is a parallelogram.   [10]

28 Prove that if in a right triangle the altitude is drawn upon the hypotenuse

    a the two triangles thus formed are similar to the given triangle and similar to each other [7]

    b each leg of the given triangle is the mean proportional between the hypotenuse and the projection of that leg on the hypotenuse   [3]

29 In triangle ABC, sides AB and AC are equal. A line through B intersects AC at D. BD is extended through D to point E, and CE is drawn. Prove that BE is greater than CE.   [10]

30 Parallel lines r and s are d distance apart and point P is any point between the two lines.

    a What is the length of the radius of a circle that is tangent to both r and s?   [1]

    b What is the locus of the center of a circle that is tangent to both r and s?   [2]

    c What is the locus of the center of a circle whose radius is $\dfrac{d}{2}$ and which passes through P?   [2]

    d On your answer paper draw the figure at the right. [*Construction* of r parallel to s is not required.] Now construct a circle tangent to the two parallel lines and passing through the given point P.   [4]

    e How many different circles are there that satisfy the requirements specified in part d?   [1]

r

$.P$

s

[1]
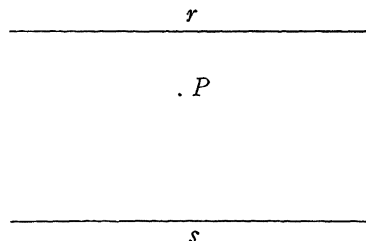
*31 The equations of four lines that intersect so as to form a quadrilateral are $y = x$, $y = x - 8$, $y = 0$ and $y = 4$.

　　　a Using graph paper, draw the graphs of these four lines.　[8]

　　　b Find the area of the quadrilateral.　[2]

　　* This question is based on one of the optional topics in the syllabus and may be used in place of any question in either part II or part III.

Part III

Answer two questions from part III.

32 In the accompanying figure, BCD is a semicircle and AB and ED are equal quadrants. (A quadrant is a quarter of a circle.) AE is 42 inches and the diameter of the semicircle is 28 inches. Find the area of the figure. [Use $\pi = \frac{2\,2}{7}$]　[10]
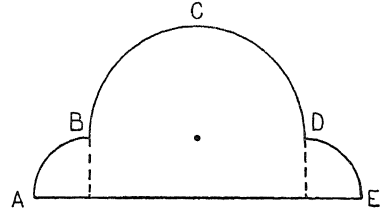
33 The longer base of an isosceles trapezoid is 40, one leg is 20 and one of the base angles is 63°.

　　a Find to the *nearest integer:*
　　　　(1) the altitude of the trapezoid　　[3]
　　　　(2) the shorter base of the trapezoid　　[4]

　　b Using the results found in answer to part *a*, find the area of the trapezoid.　[3]

34 The coordinates of the vertices of triangle ABC are $A(2, 1)$, $B(10, 1)$ and $C(6, 7)$. Find:　[10]

　　a the length of the altitude of triangle ABC on side AB
　　b the area of triangle ABC
　　c the coordinates of the mid-point of BC
　　d the length of the median to side BC [Answer may be left in radical form.]
　　e the equation of the locus of points equidistant from A and B

35 *a* Chords AB and CD of a circle intersect at E. AE is 7 inches, CE is 5 inches, and ED is 4 inches longer than EB.

　　　　(1) If the length of EB is represented by x, write an equation that can be used to find x. [2]
　　　　(2) Find the length of the longer chord.　[2]

　　b A tangent and a secant are drawn to a circle from an external point. The length of the tangent is 6 inches, and the entire secant is 5 inches longer than its external segment.

　　　　(1) If the length of the external segment is represented by x, write an equation that can be used to find x.　[3]
　　　　(2) Find the length of the entire secant.　[3]

[2]

Fill in the following lines:

Name of pupil...Name of school...

Part I

Answer all questions in part I. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 Find the sum of the interior angles of a polygon of 12 sides.

1................

2 The hypotenuse of a right triangle is 2.5 and one leg is 1.5. Find the other leg.

2................

3 The side of a square is s. Express its diagonal in terms of s.

3................

4 An angle of a rhombus is 60° and its shorter diagonal is 4. Find the altitude of the rhombus. [Answer may be left in radical form.]

4................

5 Find the area of a rhombus whose diagonals are 8 and 3.

5................

6 Find the area of a regular hexagon whose side is 2. [Answer may be left in radical form.]

6................

7 The circumference of circle O is 18π and minor arc AB of the circle is 4π. Find the number of degrees in central angle AOB.

7................

8 The locus of points equidistant from two concentric circles whose radii are 8 and 14 is a circle concentric with the given circles. Find its radius.

8................

9 Is there a regular polygon such that each exterior angle is 50°? [Answer *yes* or *no*.]

9................

10 In triangle ABC a line parallel to AC intersects AB at D and CB at E. If $AB = 12, DB = 8$, and $AC = 15$, find DE.

10................

11 In triangle ABC, angle $C = 90°$, $AC = 20$ and $BC = 36$. Find angle A to the *nearest degree*.

11................

Directions (12–15): Write on the line at the right of *each* question the expression that, when inserted in the blank, will make the statement true.

12 If line a is perpendicular to line b and line c is parallel to a, then c is ... to b.

12................

13 An angle formed by two chords intersecting within the circle is measured by one half the ... of the intercepted arcs.

13................

[3]   [OVER]

14 If the center of the circle that is circumscribed about a triangle is outside the triangle, the triangle is ....

14................

15 If two parallelograms have equal bases, their areas are to each other as their ....

15................

Directions (16–21) : For *each* of the following, if the statement is *always* true, write the word *true* on the line at the right; if it is *not always* true, write the word *false*.

16 It is always possible to construct a right triangle if the given parts are the hypotenuse and one of the acute angles.

16................

17 An equilateral polygon inscribed in a circle is regular.

17................

18 If the diagonals of a parallelogram are equal, the parallelogram is a square.

18................

19 AB and $A'B'$ are bases of isosceles triangles ABC and $A'B'C'$. If $AB : A'B' = AC : A'C'$, the triangles are similar.

19................

20 If the sides of a triangle are a, b and c, the perimeter of the triangle formed by joining the mid-points of the sides of the given triangle is
$$\frac{a + b + c}{2}$$

20................

21 The median drawn to the hypotenuse of a right triangle divides the triangle into two isosceles triangles.

21................

Directions (22–24) : Indicate the correct completion for *each* of the following by writing on the line at the right the letter a, b or c.

22 The equation $y = 4$ represents the locus of points equidistant from the two points   (a) $(0, 3)$ and $(0, 1)$ (b) $(4, 0)$ and $(0, 4)$ (c) $(0, -2)$ and $(0, 10)$
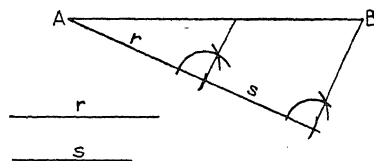
22......

23 The circle whose center is the point $(5, 0)$ and which is tangent to the y-axis passes through the point   (a) $(3, 4)$ (b) $(5, 5)$ (c) $(0, 10)$

23......

24 The distance from the point $(-1, 4)$ to the point $(6, 4)$ is   (a) 5   (b) 7   (c) 8

24......

25 The accompanying diagram shows the division of given line segment AB into two parts which are in the ratio $r : s$.

Which statement, a or b, is used to prove that the construction is correct?

    a If a line is drawn through two sides of a triangle parallel to the third side, it divides those sides proportionally.

    b A line that divides two sides of a triangle proportionally is parallel to the third side.

25.......