The University of the State of New York

317TH HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Tuesday, January 20, 1953 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in tenth year mathematics, (*c*) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year.

Part II

Answer three questions from part II.

26 Prove that if two sides of a triangle are equal, the angles opposite these sides are equal. [10]

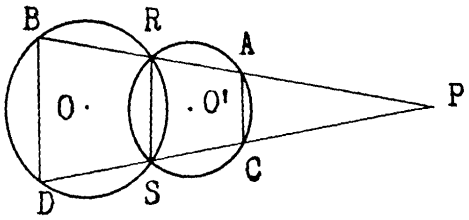
27 Tangents drawn to a circle at points A and B intersect at point P. Prove that the line from P to the center of the circle passes through the mid-point of minor arc AB. [10]

28 Prove that the area of a triangle is equal to one half the product of a side and the altitude drawn to that side. [10]

29 In the accompanying figure circles O and O' intersect at points R and S. Secants from point P through R and S intersect the circles at points A and B, and C and D respectively. Chords AC, RS and BD are drawn.

a If the number of degrees in arc RBD is represented by $2x$, express in terms of x the number of degrees in $\angle DSR$, $\angle DBR$, $\angle RSC$ and $\angle CAR$. [4]

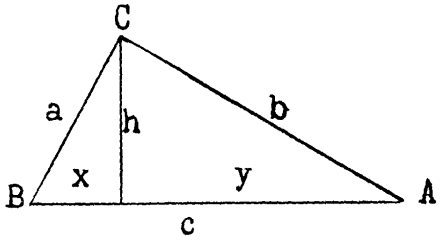
b Prove that $PA : PB = PC : PD$. [6]



30 In the accompanying figure a, b and c represent the sides of triangle ABC; x and y represent the segments made by the altitude h on side AB; also $x : h = h : y$.

Explain why each of the following statements is true:

(1) $a^2 = h^2 + x^2$ and $b^2 = h^2 + y^2$ [2]
(2) $a^2 + b^2 = x^2 + 2h^2 + y^2$ [2]
(3) $a^2 + b^2 = x^2 + 2xy + y^2$ [2]
(4) $a^2 + b^2 = c^2$ [2]
(5) ABC is a right triangle. [2]



[OVER]

Part III

Answer two questions from part III.

31 *a* If R represents the radius of the circle circumscribed about a regular pentagon, show that its apothem is equal to $R \cos 36°$ and its perimeter is equal to $10\ R \sin 36°$. [2, 2]

b Find to the *nearest integer* the apothem and the perimeter of the pentagon when $R = 5$. [2, 2]

c Using the results found in answer to *b*, find the area of the pentagon. [2]

32 The vertices of a triangle are $A\ (4, 1)$, $B\ (10, 1)$ and $C\ (8, 7)$.

a Write the equation of the locus of points equidistant from A and B. [2]

b Write the equation of the line through C parallel to AB. [2]

c Write the coordinates of the point of intersection of the lines in *a* and *b*. [2]

d Find the coordinates of the mid-point of AC and the distance from this point to B. [2, 2]

33 *a* Prove that the quadrilateral whose vertices are $A\ (4, 2)$, $B\ (7, 6)$, $C\ (10, 2)$ and $D\ (7, -2)$ is equilateral and hence is a rhombus. [6]

b Find the area of the rhombus. [2]

c Find the altitude of the rhombus. [2]

34 *a* The perimeter of a rectangle is p and its base is b.

(1) Express its altitude in terms of p and b. [2]

(2) Express its area in terms of p and b. [1]

(3) Tell whether the following statement is *true* or *false:* All rectangles which have equal perimeters have equal areas. [1]

b The bases of a trapezoid are b and b' and its altitude is h.

(1) Express its area in terms of b, b' and h. [1]

(2) If b is increased by an amount x and b' is decreased by the same amount, is the area of the trapezoid increased, is it decreased or does it remain the same? [1]

(3) If b and b' remain the same but h is increased by an amount x, show that the area is increased by $\frac{x\ (b + b')}{2}$ [4]

*35 Given line AB which passes through the points $(-2, 4)$ and $(6, 8)$.

a Find the slope of line AB. [4]

b Tell whether each of the following is *true* or *false:*

(1) Line AB passes through the point $(0, 5)$. [2]

(2) The tangent of the acute angle which the line AB makes with the x-axis is equal to $\frac{1}{2}$. [2]

(3) The equation of the line through the origin and parallel to AB is $x - 2y = 0$. [2]

* This question is based upon one of the optional topics in the syllabus.

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 Find the diagonal of a square whose side is 5. [Answer may be left in radical form.] 1.....

2 Find the area of an equilateral triangle whose side is 4. [Answer may be left in radical form.] 2.....

3 The hypotenuse of a right triangle is twice the shorter leg. Find the number of degrees in the smallest angle of the triangle. 3.....

4 Find the circumference of a circle whose radius is $3\frac{1}{2}$. [Use $\pi = \frac{22}{7}$.] 4.....

5 A side of a regular hexagon inscribed in a circle is 5. Find the area of the circle. [Answer may be left in terms of π.] 5.....

6 Two angles are complementary and one is 20° larger than the other. Find the number of degrees in the smaller angle. 6.....

7 Angles A and B of parallelogram $ABCD$ are in the ratio 4:5. Find the number of degrees in angle A. 7.....

8 Find the number of degrees in each exterior angle of a regular polygon of 10 sides. 8.....

9 Quadrilateral $ABCD$ is inscribed in a circle and its diagonals intersect at E. If arc $AB = 80°$ and arc $CD = 60°$, find the number of degrees in angle AEB. 9.....

10 In triangle ABC, D is a point on AB, E is a point on AC, and line DE is drawn. If $AB = 8$, $AC = 12$, $DB = 3$ and $EC = 4$, is DE parallel to BC? [Answer *yes* or *no*.] 10.....

11 The altitude on the hypotenuse of a right triangle is 4 and one of the segments of the hypotenuse made by the altitude is 2. Find the other segment of the hypotenuse. 11.....

12 The areas of two similar triangles are in the ratio 9:25. If an altitude of the smaller triangle is 4, find the corresponding altitude of the larger triangle. 12.....

13 The end points of the line segment AB are A (—3, 5) and B (7, 3). Find the coordinates of the mid-point of AB. 13.....

14 The number of degrees in angles A, B and C of triangle ABC are represented by $2x$, $x + 30$ and $3x$ respectively. How many degrees are there in the largest angle? 14.....

[OVER]

15 In acute triangle DEF, $DE = 10$ and angle $D = 42°$. Find to the *nearest tenth* the length of the altitude drawn to side DF. 15.....

Directions (16–19) — In the case of each of the following, if the statement is *always* true, write the word *true* on the line at the right; if it is *not always* true, write the word *false*.

16 The bisectors of two supplementary adjacent angles are perpendicular to each other. 16.....

17 If two circles whose centers are O and O' intersect at points A and B, chord AB is the perpendicular bisector of line segment OO'. 17.....

18 The bisector of an angle of a triangle bisects the side opposite this angle. 18.....

19 Two isosceles triangles are similar if they have any angle of one equal to the corresponding angle of the other. 19.....

Directions (20–23) — Indicate the correct completion for each of the following by writing the letter a, b or c on the line at the right.

20 A converse of a proposition is (*a*) always true (*b*) sometimes true (*c*) never true 20.....

21 The locus of the centers of circles tangent to a given line at a given point on that line is a (*a*) point (*b*) straight line (*c*) circle 21.....

22 Tangent PA and secant PBC are drawn to a circle from external point P. If chords AB and AC are drawn, triangle (*a*) PAB is similar to triangle PAC (*b*) PAC is similar to triangle ABC (*c*) PAB is similar to triangle ABC 22.....

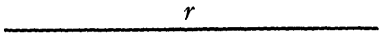
23 Two chords intersect within a circle. The segments of one chord are 5 and 12 and the segments of the other chord are x and $x + 4$. An equation that can be used to find x is (*a*) $2x + 4 = 17$ (*b*) $x^2 + 4x = 60$ (*c*) $\frac{x}{x+4} = \frac{5}{12}$ 23.....

24 The accompanying figure shows the usual method of constructing an angle equal to a given angle. Which *two* of the following statements are used to prove that angle A' is equal to angle A?

(1) Two triangles are congruent if two sides and the included angle of one are equal respectively to two sides and the included angle of the other.

(2) Two triangles are congruent if the three sides of one are equal respectively to the three sides of the other.

(3) Corresponding parts of congruent triangles are equal. 24.....



25 Given parallel lines r and s. Construct a line parallel to r and s and midway between them. [Leave all construction lines on your paper.]