The University of the State of New York

314TH HIGH SCHOOL EXAMINATION

TENTH YEAR MATHEMATICS

Monday, January 21, 1952 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II, III and IV (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in tenth year mathematics, (*c*) author of textbook used.

The minimum time requirement is four or five recitations a week for a school year.

Part II

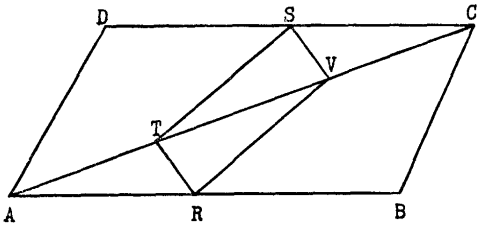
Answer two questions from part II.

26 Prove *either a* or *b:*

a The area of a trapezoid is equal to one half the product of its altitude and the sum of its bases. [10]

b The area of a regular polygon is equal to one half the product of its perimeter and its apothem. [10]

27 In parallelogram $ABCD$, R and S are the mid-points of sides AB and CD respectively. On diagonal AC, points T and V are taken so that $AT = CV$. Line segments SV, VR, RT and TS are drawn. Prove that $SVRT$ is a parallelogram. [10]



28 In isosceles triangles ABC and $A'B'C'$, AB and $A'B'$ are corresponding bases, BD and $B'D'$ are corresponding altitudes and angles DBA and $D'B'A'$ are equal. Prove

a angle BAC = angle $B'A'C'$ [3]

b $BD : B'D' = AC : A'C'$ [7]

Part III

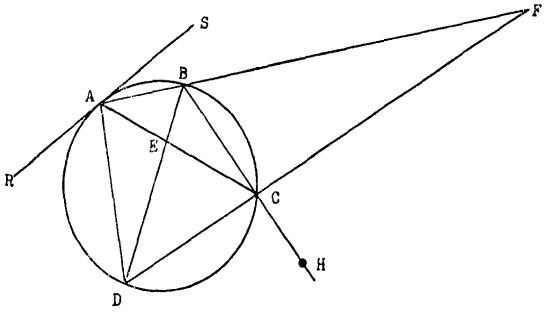
Answer two questions from part III.

29 In the accompanying figure $ABCD$ is a quadrilateral inscribed in the circle. Diagonals AC and BD intersect at E and sides AB and DC, extended, meet at F. RS is tangent to the circle at A and H is a point on BC extended. Arc $AB = 50°$, and arcs BC, CD and DA are represented by $x°$, $(x + 28)°$ and $(2x - 38)°$ respectively.

Find the number of degrees in

a arcs BC, CD and DA [3, 1, 1]

b angles AED, AFD, RAD and DCH [1, 1, 1, 2]



30 *a* CD is the altitude on the hypotenuse of right triangle ABC. $AC = 10$ and $AD = 6$. If BD is represented by x, write an equation that can be used to find x. Solve this equation for x. [3, 1]

b Chords AB and CD of a circle intersect at E. $AE = 12$ inches, $EB = 5$ inches and CE is 4 inches longer than ED. If ED is represented by x, write an equation that can be used to find x. Solve this equation for the positive value of x. [3, 3]

31 Using graph paper, draw triangle ABC whose vertices are $A(1, 2)$, $B(9, 8)$ and $C(3, 11)$.

a Find the area of triangle ABC. [6]

b Find the length of side AB. [3]

c Find the length of the altitude on AB from C. [1]

Part IV

Answer one question from part IV.

32 A side of a rhombus is 8.0 inches and an angle of the rhombus is 64°.

a Find to the *nearest tenth of an inch* each diagonal of the rhombus. [4]

b Using the results found in answer to *a*, find to the *nearest square inch* the area of the rhombus. [2]

c Check the result found in *b* by finding the altitude of the rhombus and then finding the area of the rhombus by using the formula for the area of a parallelogram. [4]

33 Each of the following statements may be completed correctly by *two* and *only two* of the four choices given. Write on your answer paper the letters *a* through *e* and after *each* letter write the *numbers* of the *two* correct completions. [10 credits — 1 credit for each correct answer].

a Two polygons of the same number of sides are similar if (1)they are mutually equiangular (2)the sides of one are proportional to the sides of the other (3)they are regular (4)they are similar to the same polygon

b If a side of a regular pentagon is represented by $2s$, the apothem by r and the radius of the circumscribed circle by R, then (1)$R = s \cos 54°$ (2)$s = R \sin 36°$ (3)$r = s \tan 36°$ (4)$r = R \sin 54°$

c Triangles ABC and $A'B'C'$ are always congruent if (1)$\angle A = \angle A'$, $\angle B = \angle B'$ and $\angle C = \angle C'$ (2)$AB = A'B'$, $\angle A = \angle A'$ and $\angle C = \angle C'$ (3)$AB = A'B'$, $BC = B'C'$ and $\angle A = \angle A'$ (4)$\angle A = \angle A'$, median CM = median $C'M'$ and altitude CD = altitude $C'D'$

d There are always four and only four points which are (1)equidistant from two given parallel lines and at a given distance from a given circle (2)equidistant from two given intersecting lines and at a given distance from their point of intersection (3)at a given distance from a given line and at a given distance from a given point (4)at a given distance from each of two given intersecting lines

e If a leg of a right isosceles triangle is represented by s, the hypotenuse by c, the perimeter by p and the area by A, then

(1) $c = \frac{s\sqrt{2}}{2}$ (2) $p = 2s + s\sqrt{2}$ (3) $s = \sqrt{2A}$ (4) $A = \frac{c^2}{2}$

*34 *a* Find the slope of line l which passes through the points A (4, 5) and B (12, 11). [4]

b *Without using graph paper* show that the point C (16, 14) lies on l. [3]

c If the point D (8, x) lies on l, find the value of x. [3]

* This question is based on one of the optional topics in the syllabus.

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2 credits. No partial credit will be allowed.

1 The legs of a right triangle are 2 and 5. Find the hypotenuse. [Answer may be left in radical form.] 1.....

2 The circumference of a circle is 10π. Find the radius. 2.....

3 The radius of a circle is 3 inches. Find to the *nearest square inch* the area of the circle. 3.....

4 Find the altitude of an equilateral triangle whose side is 8. [Answer may be left in radical form.] 4.....

5 In triangle ABC, angle $C = 90°$, angle $A = 66°$ and side $AC = 100$. Find side BC to the *nearest integer*. 5.....

6 In acute triangle ABC, side $AB = 8$ and altitude $BD = 5$. Find angle A to the *nearest degree*. 6.....

7 How many degrees are there in the sum of the interior angles of a polygon of 5 sides? 7.....

8 How many sides has a regular polygon if one of its exterior angles is 45°? 8.....

9 The areas of two similar polygons are in the ratio 1:9. Find the ratio of the perimeter of the smaller polygon to the perimeter of the larger. 9.....

10 In triangle ABC a line parallel to AC intersects AB at D and CB at E. If $AB = 8$, $BC = 12$ and $BD = 6$, find BE. 10.....

11 A tangent and a secant to a circle from an external point are 6 and 12 respectively. Find the external segment of the secant. 11.....

12 Two parallel lines are cut by a transversal and two interior angles on the same side of the transversal are represented by $x°$ and $(2x - 15)°$. Find x. 12.....

13 Write the equation of the locus of points which are on a line parallel to the x-axis and 2 units above the x-axis. 13.....

14 Find the distance from the point (2, 3) to the point (5, 7). 14.....

15 The abscissa of point A is $-2a$ and of point B is $4a$. Find the abscissa of the mid-point of line segment AB. 15.....

[OVER]

16 Given the statements:

(1) The sum of the interior angles of a triangle is one straight angle.

(2) Through a given point one and only one line can be constructed parallel to a given line.

(3) If two parallel lines are crossed by a transversal, the alternate interior angles thus formed are equal.

Which of the following represents the order in which the given statements should be arranged so that they form a logical sequence? (*a*) 3, 2, 1 (*b*) 1, 3, 2 (*c*) 2, 3, 1 (*d*) 3, 1, 2 [Answer *a*, *b*, *c* or *d*.] 16.....

17 Is statement *A* the converse of statement *B*? [Answer *yes* or *no*.]

A In a circle chords are equal if they are equidistant from the center.

B In a circle equal chords are equidistant from the center. 17.....

Directions (questions 18–23) — In *each* of the following, if the statement is *always* true, write the word *true* on the line at the right; if it is *not always* true, write the word *false*.

18 The altitudes of a triangle intersect inside the triangle. 18.....

19 The bisector of a chord of a circle passes through the center of the circle. 19.....

20 The sides of an equilateral polygon inscribed in a circle are equidistant from the center of the circle. 20.....

21 The perimeter of the triangle formed by joining the mid-points of the sides of a given triangle is one half the perimeter of the given triangle. 21.....

22 Corresponding diagonals of two similar quadrilaterals divide the quadrilaterals into triangles which are similar each to each and similarly placed. 22.....

23 If side *AB* of parallelogram *ABCD* is greater than side *BC*, angle *B* is greater than angle *A*. 23.....

Directions (questions 24–25) — Leave all construction lines on the paper.

24 Construct the locus of points equidistant from the given points *R* and *S*.

R.

.S

25 Through point *O* at the right, construct a line which makes an angle of $22\frac{1}{2}°$ with line *OA*.