The University of the State of New York

227th High School Examination

SPHERIC TRIGONOMETRY

Wednesday, June 21, 1922—1.15 to 4.15 p. m., only

Write on top of first page of answer paper (a) name of school where you have studied, (b) number of weeks and recitations a week in spheric trigonometry.

The minimum time requirement for spheric trigonometry is one recitation a week for a school year.

Answer six questions, including three from group I, one from group II and two from group III.

A, B and C represent the angles of a triangle ABC; a, b and c represent the respective opposite sides. In a right triangle, C represents the right angle.

Give special attention to neatness and arrangement of work.

In the examination in spheric trigonometry the use of the slide rule will be allowed for checking, provided all computations with tables are shown on the answer paper.

Group I

Answer three questions from this group.

1 In any right spheric triangle in which all parts except the right angle C are less than 90°, prove:

$\sin a = \sin c \sin A$ [Give geometric proof.] [8]

$\cos c = \cot A \cot B$ [7]

2 Show that in a right spheric triangle each leg and the opposite angle are always of the same species, that is, are always in the same quadrant. [15]

3 In any right spheric triangle prove the following identities:

$\sin b = \cos c \tan a \tan B$ [8]

$\sin^2 A = \cos^2 B + \sin^2 a \sin^2 B$ [7]

4 Starting with the formula

$\cos a = \cos b \cos c + \sin b \sin c \cos A$,

derive the formula $\sin \frac{A}{2} = \sqrt{\frac{\sin (s-b) \sin (s-c)}{\sin b \sin c}}$ [15]

Group II

Answer one question from this group.

5 Solve and check the quadrantal spheric triangle in which $a = 122° 40'$, $C = 65° 14'$, $c = 90°$ [12, 3]

6 Solve and check the isosceles spheric triangle in which $a = 55° 24'$, $b = 55° 24'$, $c = 70° 52'$ [11, 4]

Group III

Answer two questions from this group.

7 Solve the spheric triangle in which $a = 130° 14'$, $b = 58° 21'$, $c = 98° 16'$ [20]

8 Find the shortest distance in nautical miles between Boston (42° 21′ N., 71° 3′ W.) and Paris (48° 50′ N., 2° 21′ E.). [A nautical mile is the length of 1′ of the arc of a great circle on the earth's surface.] [20]

9 Each lateral face of a regular square pyramid makes an angle of 81° with the base. What are the face angles at a corner of the base of the pyramid? [20]