SPECIAL GEOMETRY (SMSG)
EXAMINATION

Monday, June 16, 1975 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet. Fold the last page along the perforations and, slowly and carefully, tear off the answer sheet. Then fill in the heading of your answer sheet.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL YOU ARE TOLD TO DO SO

Answer 30 questions from this part. Each correct answer will receive 2 credits. Write your answers in the spaces provided on the separate answer sheet. Where applicable, answers may be left in terms of π or in radical form.

1 If the coordinates of points R and S are $R(2,-1)$ and $S(-1,6)$, find RS.

2 In the accompanying figure, $A,B,C,$ and D are four distinct points of the circle. $\overline{AB}$ and $\overline{CD}$ intersect at P. If $AP = 4$, $PB = 16$, and $CP = PD$, find CD.

3 In the accompanying figure, P is the center of the circle, $\overline{AB}$ is a diameter, and $\overline{AC}$ and $\overline{BC}$ are chords. If $AB = 16$ and $BC = 8$, find $m\angle CAB$.

4 In a linear coordinate system, the coordinates of R and S are 2 and x, respectively. Find RS.

5 Write an equation of the straight line passing through the point $(2,3)$ and perpendicular to the x-axis.

6 Find the coordinates of the midpoint of a segment whose endpoints are $(3,-2,4)$ and $(5,0,1)$.

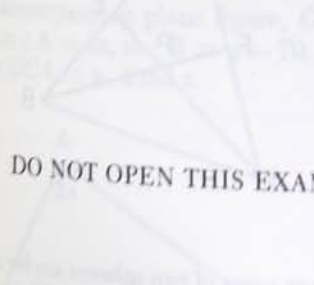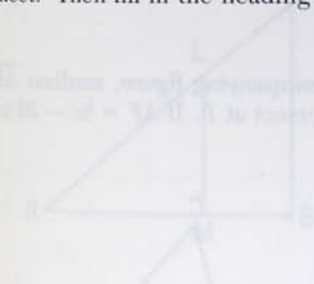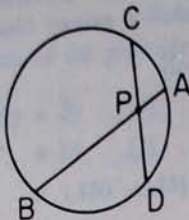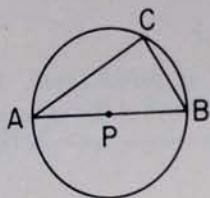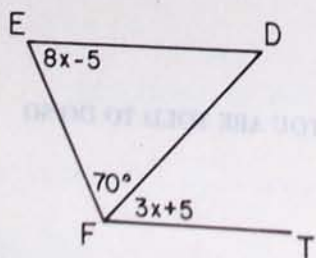7 If each angle of a regular polygon measures 144, find the number of sides of the polygon.

8 In the accompanying plane figure, $\overline{ED} \parallel \overline{FT}$, $m\angle EFD = 70$, $m\angle DFT = 3x + 5$, and $m\angle E = 8x - 5$. Find x.

9 The length of an arc of a circle is 2π and the measure of the same arc is 72. Find the radius of the circle.

10 The altitude of a right circular cone is three times the radius (r) of its base. Express the volume of the cone in terms of π and r.

11 The area of a trapezoid is 120, its altitude is 8, and one base is 18. Find the length of the other base.

12 In the accompanying plane figure, the shaded region (annulus) is formed by two concentric circles whose radii are 8 and 10. Find the area of the shaded region in terms of π.
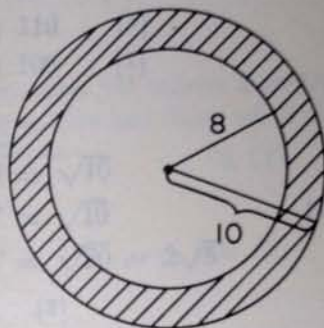
13 The bases of a right prism are equilateral triangles. Find the volume of the prism if the length of a base edge is 4 and the altitude of the prism is 10.

14 In the accompanying figure, medians $\overline{AE}$ and $\overline{BF}$ of $\triangle ABC$ intersect at P. If $AP = 8x - 20$ and $PE = 2x$, find x.
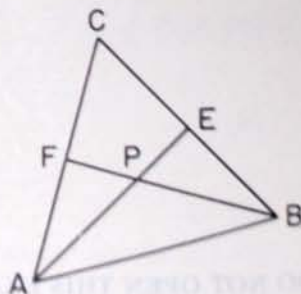
15 If the surface areas of two spheres are 9π and 25π, find the ratio of their volumes.

16 In the accompanying right triangle ABC, $\overline{CD}$ is the altitude to hypotenuse $\overline{AB}$. If $CD = 6$, $AD = x$, and $DB = x - 5$, find AD.

17 The areas of two similar triangles are 20 and 45. If a side of the smaller triangle is 4, find the corresponding side of the larger triangle.

18 Given three distinct points R, S, and T such that $RS = 3$, $ST = 4$, and $TR = 1$, what point lies between the other two?

19 Find the slope of a line perpendicular to the line passing through $(-2,3)$ and $(-3,6)$.

20 A tangent segment is drawn to a sphere of radius 8 inches from a point 17 inches from the center of the sphere. Find the number of inches in the length of the tangent segment.
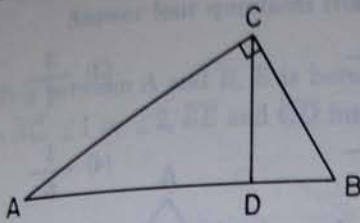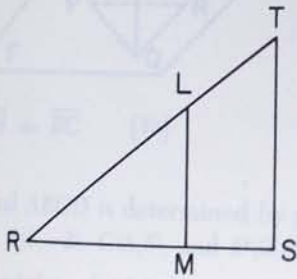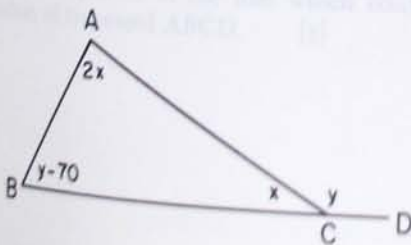
21 In the accompanying plane figure, L is between R and T, M is between R and S, $\overline{LM} \perp \overline{RS}$, $\overline{TS} \perp \overline{RS}$, $LM = 6$, $TS = 9$, and $MS = 4$. Find RM.

22 In the accompanying plane figure, C is between B and D, $m\angle A = 2x$, $m\angle B = y - 70$, $m\angle BCA = x$, and $m\angle DCA = y$. Find x.

23 A spherical weather balloon with a 10-foot radius is to be painted with a special coating requiring one quart per 100 square feet. In terms of π, how many quarts of the coating will be needed?

Directions (24–35): For *each* of those chosen, write on the separate answer sheet the *numeral* preceding the word or expression that best completes the statement or answers the question.

24 The intersection of two planes can *not* be
(1) the null set   (3) a line
(2) a point   (4) a plane

25 If $\angle A$ and $\angle B$ are complements and $m\angle A = 20 - 4x$, then $m\angle B$ may be expressed as
(1) $70 - 4x$ (3) $160 - 4x$
(2) $70 + 4x$ (4) $160 + 4x$

26 The vertices of a triangle are the points $(0,0)$, $(a,0)$, and $(0,a)$. The area of the triangle is
(1) $\dfrac{a^2}{2}$ (3) a^2
(2) $\dfrac{a^2}{4}$ (4) $a^2\sqrt{2}$

27 Lines ℓ and m are each perpendicular to plane P. Then
(1) $\overleftrightarrow{\ell}$ and $\overleftrightarrow{m}$ are skew
(2) $\overleftrightarrow{\ell}$ intersects $\overleftrightarrow{m}$
(3) $\overleftrightarrow{\ell}$ and $\overleftrightarrow{m}$ are coplanar
(4) $\overleftrightarrow{\ell}$ and $\overleftrightarrow{m}$ lie in plane P

28 In $\triangle ABC$, median AM is drawn. Then $\triangle ACM$ and $\triangle ABM$ must be
(1) congruent   (3) similar
(2) right   (4) equal in area

29 In a plane, the set of points equally distant from the points $(1,2)$ and $(1,8)$ is the line whose equation is
(1) $y = 5$ (3) $x = 1$
(2) $x = 5$ (4) $x + y = 5$

30 The distance between points A and B is 8. The set of all points, P, in space such that $AP = BP = 5$ is a
(1) line segment   (3) line
(2) circle   (4) pair of points

31 In $\triangle ABC$, if $AB = BC$ and $m \angle A = 50$, then

(1) $AB > AC$
(2) $BC > AC$
(3) $AC > AB$
(4) $AC = AB$

32 If a law states "If a man is 80 years of age or older, then he cannot drive a car," which must necessarily be true?

(1) Jim drives a car; therefore he is younger than 80.
(2) Bill does not drive a car; therefore he is younger than 80.
(3) Steve is over 80; therefore Steve may drive a car.
(4) Tom is younger than 80; therefore Tom drives a car.

33 If $c > 2$, then $|c - 2|$ is equivalent to

(1) $c + 2$
(2) $c - 2$
(3) $-(c + 2)$
(4) $-(c - 2)$
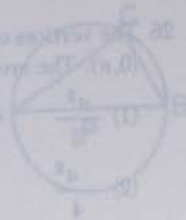
34 In a circle, a central angle which measures m intercepts an arc which measures $3y$ and an inscribed angle which measures t intercepts an arc which measures y. The ratio $\dfrac{t}{m}$ equals

(1) $\dfrac{1}{3}$
(2) $\dfrac{6}{1}$
(3) $\dfrac{3}{1}$
(4) $\dfrac{1}{6}$
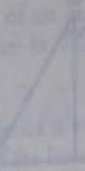
35 The two-dimensional graph of the set of points such that $y < 1$ or $y > 3$ is

(1) a horizontal strip
(2) a vertical strip
(3) two nonintersecting half-planes
(4) the intersection of two half-planes

Answers to the following questions are to be written on paper provided by the school.
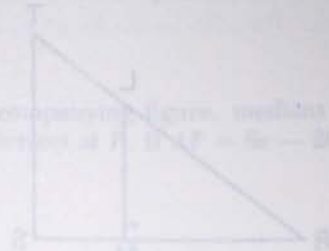
Part II

Answer four questions from this part. Show all work unless otherwise directed.

36 Given: D is between A and B, E is between A and C, $\overline{AB} \cong \overline{AC}$, $\angle 1 \cong \angle 2$, $\overline{BE}$ and $\overline{CD}$ intersect at P.
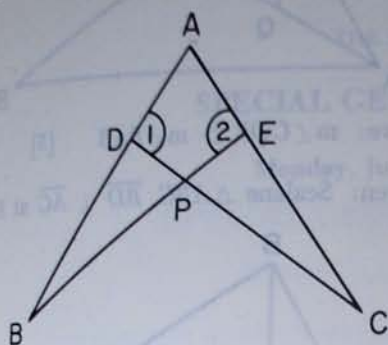
Prove: a $\angle B \cong \angle C$ [4]

     b $\overline{BP} \cong \overline{CP}$ [6]

37 Given: Points A, B, and C in plane E, plane $E \parallel$ plane F, $\overline{AP} \perp F$ at P, $\overline{BQ} \perp F$ at Q, $\overline{CR} \perp F$ at R, $\overline{PQ} \cong \overline{QR}$.

Prove: $\overline{AB} \cong \overline{BC}$ [10]

38 Quadrilateral $ABCD$ is determined by points $A(-4,-6)$, $B(6,-2)$, $C(8,5)$, and $D(0,8)$.

a Draw quadrilateral $ABCD$.   [1]

b Show that $ABCD$ is a trapezoid and give a reason for your conclusion.   [3]
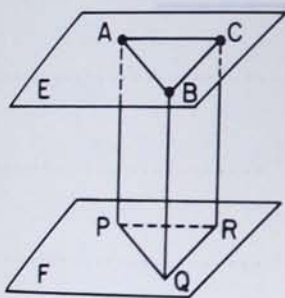
c Find the coordinates of the endpoints of the median of trapezoid $ABCD$.   [3]

d Write an equation of the line which contains the median of trapezoid $ABCD$.   [3]

39 In the plane figure, $\triangle ABC$ is inscribed in a circle, $\overline{SC}$ is tangent to the circle at C, $\overline{SB}$ bisects $\angle B$, $m\widehat{AB} = 3x + 10$, $m\widehat{BC} = 2x$, and $m\widehat{CA} = x + 50$.
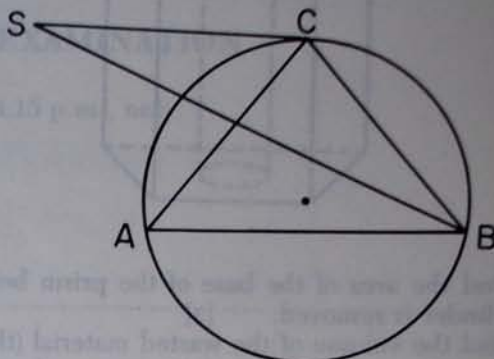
a Find x.   [2]

b Show that $\triangle ABC$ is isosceles and give a reason for your conclusion.   [3]

c Find $m \angle S$.   [3]

d Find $m \angle SCA$.   [2]

40 Given: $\overline{AB}$ is a diameter of the circle, $\overline{CD}$, $\overline{CB}$, and $\overline{DB}$ are chords, $\overline{AB} \perp \overline{CD}$ at E.
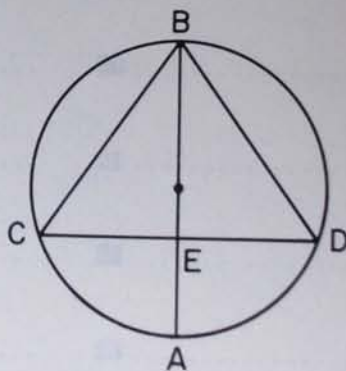
Prove: $EB \times AB = CB \times DB$ [10]

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41 As shown in the diagram, a piece of jewelry is to be made from a solid piece of gold in the shape of a right prism through which a cylindrical hole has been bored. The base of the prism is an isosceles trapezoid whose parallel sides are 9 millimeters and 15 millime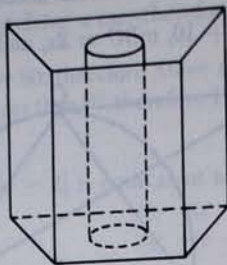ters and whose base angles measure 45. The height of the prism is 12 millimeters and the diameter of the hole is 4 millimeters.

a Find the area of the base of the prism before the cylinder is removed.   [3]

b Find the volume of the wasted material (the cylinder). [*Answers should be left in terms of π.*]   [3]

c Find the volume of the piece of jewelry. [*Answers should be left in terms of π.*]   [4]

42 Choose *two* parts from *a*, *b*, and *c*.

a Given: $\triangle ABC$, $\overline{AP}$ bisects $\angle CAB$, P is between B and C, Q is between A and P.
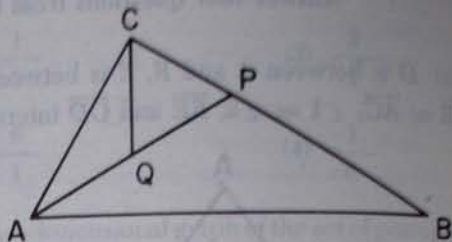
Prove: $m \angle CQP > m \angle PAB$ [5]

b Given: Scalene $\triangle ABC$, $\overline{BD} \perp \overline{AC}$ at D
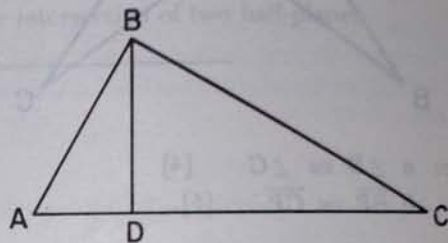
Prove: $AD \neq DC$ [5]

c Write the equation of a sphere with center at the origin in a 3-dimensional coordinate system and which passes through the point $(2,3,-4)$.   [5]

The University of the State of New York

THE STATE EDUCATION DEPARTMENT

SPECIAL GEOMETRY (SMSG) EXAMINATION

Monday, June 16, 1975 — 1:15 to 4:15 p.m., only

ANSWER SHEET

Pupil .... Teacher ....
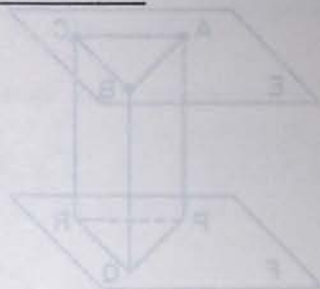
School ....

Your answers to Part I should be recorded on this answer sheet.

Part I

Answer 30 questions in this part.

1........................	10	19 ....
2........................	11	20 ....
3........................	12	21 ....
4........................	13	22 ....
5........................	14	23 ....
6........................	15	24 ....
7........................	16	25 ....
8........................	17	26 ....
9........................	18	27 ....

Questions 28 through 35 should be answered on the back of this page.

SMSG-6-'75

[OVER]

28 ....

29 ....

30 ....

31 ....

32 ....

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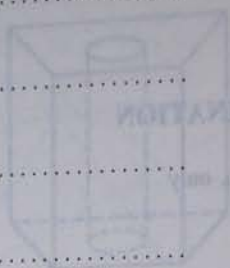34 ....
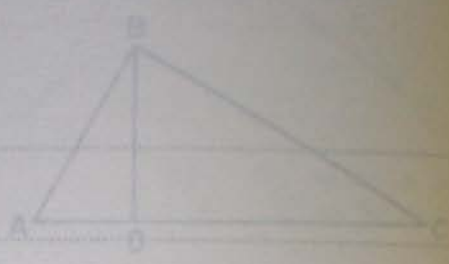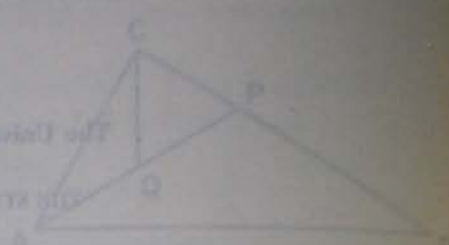
35 ....

Your answers for Part II should be placed on paper provided by the school.

FOR TEACHERS ONLY

The University of the State of New York

SCORING KEY

SPECIAL GEOMETRY (SMSG) EXAMINATION

Monday, June 16, 1975 — 1:15 to 4:15 p.m., only

Unless otherwise specified, mathematically correct variations in the answers will be allowed.

Part I

Allow a total of 60 credits, 2 credits for each of 30 of the following: [If more than 30 are answered, only the first 30 answered should be considered.]

(1) $\sqrt{58}$	(13) $40\sqrt{3}$	(25) 2
(2) 16	(14) 5	(26) 1
(3) 30	(15) $^{27}/_{125}$	(27) 3
(4) $\lvert 2 - x \rvert$ or $\lvert x - 2 \rvert$	(16) 9	(28) 4
(5) $x = 2$	(17) 6	(29) 1
(6) $(4, -1, ^5/_2)$	(18) R	(30) 2
(7) 10	(19) $^1/_3$	(31) 3
(8) 10	(20) 15	(32) 1
(9) 5	(21) 8	(33) 2
(10) πr^3	(22) 35	(34) 4
(11) 12	(23) 4π	(35) 3
(12) 36π	(24) 2	

38 *c* $(1, -4), (4, 6^{1}/_{2})$ [3]
  d $y = {}^{7}/_{2}\,x - {}^{15}/_{2}$ [3]

39 *a* 50 [2]
  c 25 [3]
  d 50 [2]

41 *a* 36 [3]
  b 48π [3]
  c $432 - 48\pi$ [4]

42 *c* $x^2 + y^2 + z^2 = 29$ [5]