The University of the State of New York

EXAMINATION FOR QUALIFYING CERTIFICATES

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SOLID GEOMETRY

Monday, September 14, 1925 — 1.15 to 4.15 p. m., only

Answer eight questions. Irrational results may be left in the form of π and radicals unless otherwise stated. Papers entitled to less than 75 credits will not be accepted.

1 Prove that if two straight lines are cut by three parallel planes, the corresponding segments are proportional.

2 Prove that in two polar triangles each angle of one is measured by the supplement of the side lying opposite to it in the other.

3 Prove that the volumes of similar cylinders of revolution are to each other as the cubes of the altitudes or as the cubes of the radii of the bases.

4 Complete and prove the following: The volume of a triangular prism is equal to . . .

5 The base of a quadrangular pyramid is a rhombus of which the side is 29 and one diagonal 42. If the altitude of the pyramid is 28, find its volume.

6 If through each of two parallel lines a plane is passed and these two planes intersect, prove that their intersection is parallel to each of the given lines.

7 *a* The altitude of the torrid zone is 3200 miles. Find its area if the earth is assumed to be a sphere with a radius of 4000 miles.

b If a solid iron ball 4 inches in diameter weighs 9 pounds, find the weight of a spheric shell of the same material 1 inch thick, if its outer diameter is 8 inches.

8 *a* Find the locus of all lines drawn from a point in a plane and making a given angle with the plane.

b Find the locus of points equidistant from three points in space, the three points not all lying in one straight line.

9 The altitude of a right circular cone is 16 inches and the radius of its base is 12 inches. Four inches from the vertex a plane is passed parallel to the base. Find the lateral area of the frustum thus formed.

10 Prove that the projections on a plane of equal and parallel lines are equal and parallel.

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9 *a* If the radius of a sphere is increased 50%, by what per cent is its volume increased? By what per cent is its surface increased? [3, 3]

b Show that the area of a trirectangular spheric triangle on the surface of a sphere is equal to one eighth of the area of the sphere. [6½]

10 Given a sphere whose radius is 2″ and a fixed line through its center; find the locus of points in space 3″ from the surface of the sphere and 3″ from the line. [5, 5, 2½]

11 A watering trough in the form of a regular triangular prism has one of its lateral edges on the ground and the opposite face horizontal. It is filled with water to a depth of two thirds of the altitude of the trough in this position. What part of the volume of the trough is occupied by the water? [12½]

12 State whether *each* of the following statements is true or false: [Label each answer with the corresponding letter.]

a If a right circular cylinder circumscribes a sphere, the lateral area of the cylinder is greater than the surface of the sphere. [2½]

b Three planes tangent to the same sphere can not be parallel. [2]

c Three planes parallel to the same line are parallel to each other. [2]

d Three planes may have only one point in common. [2]

e Two faces of a truncated prism may be parallel. [2]

f Any face angle of a trihedral angle is greater than the difference between the other two face angles. [2]