The University of the State of New York

EXAMINATION FOR QUALIFYING CERTIFICATES

SOLID GEOMETRY

Monday, September 12, 1921—1.15 to 4.15 p. m., only

Answer eight questions, including not more than four from group I. Papers entitled to less than 75 credits will not be accepted.

Group I

Answer not more than four questions from this group.

1 Prove that if a line is perpendicular to each of two lines at their point of intersection, it is perpendicular to the plane of these lines.

2 Prove that the sum of the face angles of any convex polyhedral angle is less than four right angles.

3 Prove that the volume of any triangular pyramid is equal to one third of the product of its base and its altitude.

4 Prove that every section of a sphere made by a plane is a circle.

5 Prove that in two polar triangles, each angle of one is the supplement of the side lying opposite to it in the other.

Group II

6 *a* How is the angle between the sides of a spheric triangle measured?

b How is the angle between two intersecting planes measured?

c Prove that all plane angles of the same dihedral angle are equal.

7 Prove that the two planes determined by two parallel lines and a point not in their plane, intersect in a line which is parallel to each of the given parallels.

8 Prove that the diagonals of any rectangular parallelepiped are equal.

9 A cube is inscribed in a sphere. Find the ratio of their volumes and of their surfaces.

Group III

Irrational results may be left in the form of π and radicals unless otherwise stated.

10 A lead pencil whose ends are regular hexagons was cut from a cylindric piece of wood with the least waste of wood. If the original piece was 8 inches long and $\frac{1}{2}$ of an inch in diameter, find the volume of the pencil.

11 If the radius of a sphere is 2.5 inches and the sides of a triangle on this sphere are 104°, 115° and 101°, find the area of the polar triangle.

12 The slant height of a regular pyramid is 39 feet, the altitude is 36 feet and the base is a square. Find the lateral area and the volume.