

The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, August 21, 1958 — 12 m. to 3 p.m., only

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Name of pupil.....Name of school.....

Name and author of textbook used.....

Part I

Answer all questions in this part. Each correct answer will receive  $2\frac{1}{2}$  credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of  $\pi$  or in radical form.

- 1 Find the diagonal of a rectangular solid whose edges are  $2\sqrt{3}$ , 3 and 2. 1.....
- 2 Find the total area of a regular tetrahedron whose edge is 8. 2.....
- 3 The volumes of two similar prisms are in the ratio 8:125. Find the ratio of the lateral area of the smaller prism to that of the larger prism. 3.....
- 4 The total area of a cube is 54. Find the area of the inscribed sphere. 4.....
- 5 Find the number of spherical degrees in the area of a spherical triangle whose angles are  $120^\circ$ ,  $130^\circ$  and  $140^\circ$ . 5.....
- 6 A zone is drawn on a sphere whose radius is 6. If the altitude of the zone is 3, find its area. 6.....
- 7 A lune whose angle is  $40^\circ$  lies on a sphere whose radius is 3 inches. Find the number of square inches in the area of the lune. 7.....
- 8 A right triangle whose legs are 3 and 4 is revolved through  $360^\circ$  about its shorter leg as an axis. Find the volume of the solid generated. 8.....
- 9 The edges of the bases of the frustum of a regular square pyramid are 4 and 7 and its slant height is 5. Find the lateral area of the frustum. 9.....
- 10 A line 20 inches long is inclined to a plane at an angle of  $41^\circ$ . Find, to the nearest tenth of an inch, the number of inches in the length of its projection on the plane. 10.....

SOLID GEOMETRY — *continued*

- 11 The slant height of a regular square pyramid is 13 and its altitude is 12. Find the cosine of the dihedral angle formed by the base and a lateral face. 11.....
- 12 If the lateral area of a right circular cylinder is equal numerically to its volume, find the radius of the base. 12.....
- 13 The slant height of a right circular cone is 15 and its lateral area is  $135\pi$ . Find the radius of the base. 13.....
- 14 A cone is cut into two parts by a plane parallel to the base and passing through the midpoint of the altitude. Find the ratio of the volume of the smaller part to the volume of the given cone. 14.....
- Directions (15-20):* Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b* or *c*.
- 15 Two face angles of a trihedral angle are  $120^\circ$  and  $150^\circ$ . The third face angle is (a)acute (b)right (c)obtuse 15.....
- 16 An equilateral spherical triangle may have each of its sides equal to (a)  $150^\circ$  (b)  $120^\circ$  (c)  $90^\circ$  16.....
- 17 A line parallel to the intersection of two planes is (a)oblique to one of the planes (b)parallel to one of the planes (c)perpendicular to one of the planes 17.....
- 18 The locus of points equidistant from three noncollinear points is a (a)point (b)line (c)plane 18.....
- 19 Plane *A* is oblique to plane *B*. A polygon *P* in plane *A* is projected onto plane *B*. If the projection on plane *B* is a square, polygon *P* may be a (a)rectangle (b)square (c)quadrilateral which is not a parallelogram 19.....
- 20 A regular polyhedron whose faces are equilateral triangles may have (a)six faces (b)eight faces (c)twelve faces 20.....

**Part II**

**Answer three questions from this part.**

- 21 Prove: If two planes are perpendicular to each other, a line drawn in one of them perpendicular to their intersection is perpendicular to the other. [10]
- 22 In tetrahedron  $V-ABC$ , a plane is passed parallel to the opposite skew edges  $VC$  and  $AB$ , cutting the tetrahedron in the section  $RSTW$ . Prove that  $RSTW$  is a parallelogram. [10]
- 23 Describe fully the locus of points *on the surface of a sphere* that are  
*a* equidistant from two fixed points on the sphere [3]  
*b* equidistant from the center of the sphere and a fixed point on the sphere [3]  
*c* equidistant from two fixed diameters [4]
- 24 Prove: A spherical angle is measured by the arc of the great circle described from its vertex as a pole and included between its sides produced if necessary. [10]
- 25 Plane  $M$  is given parallel to plane  $P$ . Three solids are placed as follows:  
*a* a sphere, radius  $r$ , center on  $M$ , tangent to  $P$   
*b* a circular cone, radius  $r$ , vertex on  $M$ , base on  $P$   
*c* a circular cylinder, radius  $r$ , one base on  $M$ , one base on  $P$   
 A plane  $S$  is passed between  $M$  and  $P$  and parallel to them at a distance  $d$  from  $M$ , intersecting the three solids. Show that the area of the section of the cylinder is equal to the sum of the area of the section of the sphere and the area of the section of the cone. [10]

Part III

Answer two questions from this part. Show all work.

- 26 A container in the shape of a right circular cylinder, radius 8 inches, is partly filled with water. A spherical solid, radius 6 inches, is dropped into the container in such a way that when the sphere rests on the bottom of the container, the level of the water rises until it just reaches the top of the spherical solid. What was the height of the water in the container before the spherical solid was dropped into it? [10]

- 27 Two angles of an isosceles spherical triangle are  $52^\circ$  and  $76^\circ$ . The area of the triangle is  $30\pi$ . Find

*a* the third angle of the triangle [3]

*b* the area of the sphere [3]

*c* the volume of the sphere [4]

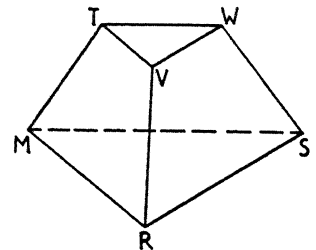
[Answers may be left in terms of  $\pi$ .]

- 28 The volume of a rectangular solid whose edges are in the ratio 1:2:3 is equal to the volume of a circular cone whose radius is  $r$  and whose altitude is  $h$ .

*a* If  $x$  represents the shortest edge of the rectangular solid, express  $x$  in terms of  $r$  and  $h$ . [5]

*b* If  $r = 15$  and  $h = 12$ , find  $x$  to the nearest hundredth. [5]

- 29 In the figure at the right,  $MRS-TVW$  is a frustum of a triangular pyramid.  $K$  is the area of  $\triangle MRS$ ,  $G$  is the area of  $\triangle TVW$  and  $a$  is the altitude of the frustum. A plane is passed through  $MVS$  and a plane is passed through  $MVW$ . Show that the volume of the pyramid  $V-MWS$  is  $\frac{a}{3} \sqrt{GK}$ .



[The volume of the frustum of a pyramid is given by the formula

$$V = \frac{h}{3} (B_1 + B_2 + \sqrt{B_1 B_2}) \quad .] \quad [10]$$

# FOR TEACHERS ONLY

# SG

## INSTRUCTIONS FOR RATING SOLID GEOMETRY

Thursday, August 21, 1958 — 12 m. to 3 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

### Part I

Allow  $2\frac{1}{2}$  credits for each correct answer; allow no partial credit. For questions 15–20, allow credit if the pupil has written the correct expression instead of the letter *a*, *b* or *c*.

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|---------------------------|---------------------|
| (1) 5                     | (11) $\frac{5}{13}$ |
| (2) $64\sqrt{3}$ or 110.7 | (12) 2              |
| (3) 4:25                  | (13) 9              |
| (4) $9\pi$ or 28.3        | (14) $\frac{1}{8}$  |
| (5) 210                   | (15) <i>a</i>       |
| (6) $36\pi$ or 113.0      | (16) <i>c</i>       |
| (7) $4\pi$ or 12.6        | (17) <i>b</i>       |
| (8) $16\pi$ or 50.2       | (18) <i>b</i>       |
| (9) 110                   | (19) <i>a</i>       |
| (10) 15.1                 | (20) <i>b</i>       |