The University of the State of New York

REGENTS HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Wednesday, August 21, 1957 — 12 m. to 3 p.m., only

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Fill in the following lines:

Name of pupil...Name of school..

Name and author of textbook used..

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 The total area of a cube is 150. Find its volume.

1................

2 A rectangular parallelepiped has the dimensions 3, 6 and x, and a diagonal of 7. Find x.

2................

3 A cylinder of revolution has a lateral area of π and an altitude of $1\frac{1}{2}$. Find the radius of its base.

3................

4 The radius of the base of a cone of revolution is 3 and its slant height is 5. Find the lateral area of the cone.

4................

5 A pyramid has an altitude of 9 and a square base whose perimeter is 40. Find the volume of the pyramid.

5................

6 A cone of revolution has an altitude of 5 inches. The area of a section made by a plane parallel to the base and 2 inches from the vertex is 4 square inches. Find the number of square inches in the area of the base of the cone.

6................

7 The bases of a frustum of a regular pyramid are pentagons. A side of the upper base is 4, a side of the lower base is 6 and the slant height is 3. Find the lateral area of the frustum.

7................

8 The volumes of two similar cylinders are 1 and 64. Find the ratio of the altitude of the smaller cylinder to the altitude of the larger.

8................

[1] [OVER]

9 The area of a sphere is S. Express the area of a great circle of the sphere in terms of S.

9................

10 The altitude of a zone on a sphere of radius r is equal to one-fourth of the radius of the sphere. Express the area of the zone in terms of r.

10................

11 A line 10 inches long is inclined at an angle θ to a plane and makes a projection of 8.9 inches on the plane. Find θ to the *nearest degree*.

11................

12 The angles of a spherical triangle are 61°, 62° and 63°. Find the number of spherical degrees in the area of the triangle.

12................

Directions (13–16) : Indicate the correct completion for *each* of the following by writing on the line at the right the letter a, b, c or d.

13 The face angles of a trihedral angle may be   (*a*) 50°, 60°, 120°
(*b*) 50°, 60°, 110°   (*c*) 50°, 60°, 100°   (*d*) 110°, 120°, 130°

13................

14 If one side of a spherical triangle contains 50°, the angle opposite this side in the polar triangle contains   (*a*) 25°   (*b*) 40°   (*c*) 100°
(*d*) 130°

14................

15 The angles of a spherical triangle may be   (*a*) 40°, 50°, 60°
(*b*) 50°, 60°, 70°   (*c*) 150°, 160°, 170°   (*d*) 150°, 180°, 210°

15................

16 The distance between two points A and B is $2x$. The locus of points at a distance x from both A and B is   (*a*)a point   (*b*)a straight line
(*c*)a plane   (*d*)two spherical surfaces

16................

Directions (17–20) : For *each* of the following tell whether the statement is always true, sometimes true or never true by writing the word *always, sometimes* or *never* on the line at the right.

17 If a line is perpendicular to one of two perpendicular lines, it is perpendicular to the other.

17................

18 If a line is perpendicular to one of two intersecting planes, it is parallel to the other.

18................

19 Through a line which is oblique to plane P, one plane, and only one, can be passed perpendicular to P.

19................

20 If plane M is perpendicular to plane N and if P is a point in M but not in N, the line through P, perpendicular to N, lies in M.

20................

[2]

Part II

Answer three questions from this part.

21 Prove: If a line is perpendicular to a plane, every plane passed through the line is perpendicular to the given plane.   [10]

22 Prove: If a point on a sphere is at a quadrant's distance from each of two other points on the sphere, not the extremities of a diameter, it is the pole of the great circle passing through these points.   [10]

23 Prove: If a spherical triangle is isosceles, its polar triangle is isosceles.   [10]

24 Points A and B are in a plane P. Each locus listed in column I below is described briefly *once* and *only once* in column II. List the numbers 1–5 on your answer paper. After *each* number write the *letter* indicating the corresponding brief description of the locus.   [10]

| *Column I*	*Column II*
Locus of points	a a line perpendicular to P
(1) equidistant from A and B	b a line parallel to P
(2) at a distance d from line through A and B	c a circle
(3) at a distance m from midpoint of line segment AB	d two circles
	e a plane perpendicular to P
(4) satisfying both (1) and (2) above	f a plane parallel to P
(5) at a distance n from the locus determined in (1)	g two parallel planes perpendicular to P
	h two planes parallel to P
	i a spherical surface
	j two spherical surfaces
	k a cylindrical surface
	l two cylindrical surfaces

25 An element of a frustum of a cone of revolution makes an angle θ with the lower base. The upper base radius is r and the lower base radius is $3r$. Express the lateral area S of the frustum in terms of r and a trigonometric function of θ.   [10]

[3]   [OVER]

Part III

Answer two questions from this part. Show all work.

26 Each end of the tank shown in the figure is a half cylinder; the middle part is a rectangular parallelepiped. The tank is 90 inches long, 42 inches wide and 30 inches high. Find the capacity of the tank to the *nearest ten gallons*. [1 gallon = 231 cubic inches; use the approximation $\pi = \frac{22}{7}$.]   [10]
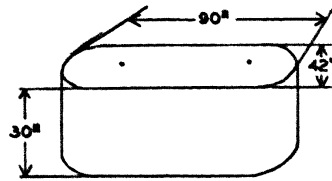
27 A platform is to be supported by ten concrete piers. Find, to the *nearest cubic yard*, the amount of concrete needed to construct the ten piers in *each* of the following cases:

a Each pier is 3 feet high and is a frustum of a regular square pyramid, the upper base edge being 3 feet and the lower base edge 4 feet. [$V = \frac{1}{3}h\,(B_1 + B_2 + \sqrt{B_1 B_2})$.]   [5]

b Each pier is 3 feet high and is a frustum of a right circular cone, the diameters of the top and bottom being 3 feet and 4 feet, respectively. [$V = \frac{1}{3}\pi h\,(r_1^2 + r_2^2 + r_1 r_2)$. Use the approximation $\pi = \frac{22}{7}$.]   [5]

28 The volume of a sphere is 2,400 cubic inches.

a Find the radius of the sphere to the *nearest tenth of an inch*.   [6]

b Using the result found in answer to *a*, find, to the *nearest square inch*, the area of a lune on the sphere whose angle is 20°. [Use the approximation $\pi = 3.14$.]   [4]

29 The area of a spherical triangle on a sphere of radius 3.5 inches is 30 square inches. Two angles of the triangle are 63° and 105°. Find the third angle of the triangle to the *nearest degree*. [Use the approximation $\pi = \frac{22}{7}$.]   [10]

[4]

FOR TEACHERS ONLY

SG
INSTRUCTIONS FOR RATING
SOLID GEOMETRY
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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow $2\frac{1}{2}$ credits for each correct answer; allow no partial credit. For questions 13–16, allow credit if the pupil has written the correct expression instead of the letter *a, b, c* or *d*.

(1) 125

(2) 2

(3) $\frac{1}{3}$

(4) 15π *or* $47\frac{1}{7}$ *or* 47.1

(5) 300

(6) 25

(7) 75

(8) 1:4

(9) $\frac{1}{3}S$

(10) $\frac{1}{2}\pi r^2$

(11) 27

(12) 6

(13) *c*

(14) *d*

(15) *c*

(16) *a*

(17) sometimes

(18) sometimes

(19) always

(20) always

Part II
Answers to the objective question only are given below.

24 (1) *e*

(2) *k*

(3) *i*

(4) *c*

(5) *g*