

The University of the State of New York

325TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Wednesday, August 24, 1955 — 12 m. to 3 p.m., only

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Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) names of schools where you have studied, (b) number of weeks and recitations a week in solid geometry previous to entering summer high school, (c) number of recitations in this subject attended in summer high school of 1955 or number and length in minutes of lessons taken in the summer of 1955 under a tutor licensed in the subject and supervised by the principal of the school you last attended, (d) author of textbook used.

The minimum time requirement is four or five recitations a week for half a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1955 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

Answer two questions from this part.

21 Prove: If each of two intersecting planes is perpendicular to a third plane, their line of intersection is also perpendicular to the given plane. [10]

22 Prove: In two polar triangles, each angle of one has the same measure as the supplement of the side lying opposite to it in the other. [10]

23 The base of a right parallelepiped is a rhombus. Prove that the plane passing through two diagonally opposite lateral edges is perpendicular to the plane passing through the other lateral edges. [10]

24 Plane  $M$  is parallel to plane  $N$ .  $A$  and  $B$  are points in  $M$ ;  $C$  is a point in  $N$ .

a Describe *fully* the locus of points

(1) equally distant from  $M$  and  $N$  [2]

(2) equally distant from  $A$ ,  $B$  and  $C$  [4]

(3) that satisfy the conditions of (1) and (2) if the plane of  $A$ ,  $B$  and  $C$  is oblique to  $M$  [2]

b Indicate the correct completion for the following statement by writing the letter  $a$ ,  $b$  or  $c$  after the number (4) on your answer paper:

(4) If line  $AC$  is perpendicular to plane  $M$ , the number of points that satisfy the conditions of (1) and (2) is (a) none (b) one (c) more than one [2]

[1]

[OVER]

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Part III

Answers three questions from this part. Show all work.

25 The concrete standard for a parking sign has the form of a right circular cylinder surmounted by a frustum of a cone of revolution. The radius of the base of the cylinder is 10 inches and the altitude of the cylinder is 3.5 inches. The slant height of the frustum is 5 inches and the radii of the bases are 8 inches and 4 inches. Determine the number of standards that can be made from 11 cubic feet of concrete, making no allowance for waste. [Use  $\pi = \frac{22}{7}$ ; the formula for the volume of a frustum of a cone of revolution is  $V = \frac{\pi h}{3} (r_1^2 + r_2^2 + r_1 r_2)$ .] [10]

26 An equilateral spherical triangle and a circle of the sphere have equal areas. The radius of the sphere is 16 inches. The plane of the circle is 8 inches from the center of the sphere. Find the number of degrees in an angle of the spherical triangle. [10]

27 A right circular cylinder is inscribed in a sphere of radius  $R$ . The altitude of the cylinder equals the diameter of its base.

*a* Show that the lateral surface of the cylinder is one half the surface of the sphere. [5]

*b* Show that the volume of the cylinder is approximately one half the volume of the sphere. [5]

28 A lateral edge of a regular hexagonal pyramid is 4 and makes with its projection on the base an angle  $\theta$ .

*a* Show that the volume of the pyramid is given by the formula  $V = 32\sqrt{3} \cos^2 \theta \sin \theta$ . [5]

*b* Using logarithms, find  $V$  to the nearest integer when  $\theta = 70^\circ$ . [5]

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Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive  $2\frac{1}{2}$  credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of  $\pi$  or in radical form.

1 The radius of the base of a right circular cone is 5 and the altitude is 12. Find the volume. 1.....

2 The total surface of a cube is 384. Find a diagonal of the cube. 2.....

3 The volume of a right prism is 210. The base is a right triangle with legs 5 and 12. Find a lateral edge of the prism. 3.....

4 The base edge of a regular square pyramid is 6. The altitude of the pyramid is 4. Find the lateral area. 4.....

5 The slant height of a regular tetrahedron is  $4\sqrt{3}$ . Find the total area of the tetrahedron. 5.....

6 The area of the base of a pyramid is 45 square inches and the altitude of the pyramid is 12 inches. A plane parallel to the base of the pyramid cuts the altitude 4 inches from the vertex. Find the number of square inches in the area of the section thus formed. 6.....

7 The slant height of a frustum of a right circular cone is 8. The radii of the bases of the frustum are 4 and 6. Find the lateral area of the frustum. 7.....

8 The volumes of two similar prisms are in the ratio 27:8. Find the ratio of the altitude of the larger prism to that of the smaller. 8.....

9 An isosceles triangle with base  $2a$  and leg  $b$  is revolved through  $180^\circ$  about the altitude to the base as an axis. Express the lateral area of the resulting solid in terms of  $a$  and  $b$ . 9.....

10 On a sphere of radius 8, the area of a zone is  $80\pi$ . Find the altitude of the zone. 10.....

11 The area of a lune is one fifth the area of the sphere. Find the number of degrees in the angle of the lune. 11.....

## SOLID GEOMETRY

*Directions (12-17):* For *each* of the following, tell whether the statement is *always* true, *sometimes* true or *never* true by writing on the line at the right the word *always*, *sometimes* or *never*.

12 If a line is perpendicular to one of two parallel lines, it is perpendicular to the other. 12.....

13 If a line is parallel to one of two intersecting planes, it is parallel to the other. 13.....

14 The projection of a line segment on a plane is greater than the line segment. 14.....

15 If a sphere is tangent to the faces of a trihedral angle, the points of contact determine a small circle of the sphere. 15.....

16 If the planes of two great circles of a sphere are perpendicular to each other, the pole of one circle lies on the other. 16.....

17 The lateral faces of a regular triangular pyramid are right triangles. 17.....

*Directions (18-20):* Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b* or *c*.

18 Two face angles of a trihedral angle are  $60^\circ$  and  $140^\circ$ . The third face angle may be (a)  $100^\circ$  (b)  $160^\circ$  (c)  $190^\circ$  18.....

19 The locus of points equidistant from two intersecting planes is (a) one plane (b) two planes (c) four planes 19.....

20 If  $E$  represents the spheric excess of a spherical triangle, the perimeter of its polar triangle equals (a)  $(540 - E)^\circ$  (b)  $(360 + E)^\circ$  (c)  $(360 - E)^\circ$  20.....

# FOR TEACHERS ONLY

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## INSTRUCTIONS FOR RATING SOLID GEOMETRY

Wednesday, August 24, 1955 — 12 m. to 3 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

### Part I

Allow  $2\frac{1}{2}$  credits for each correct answer; allow no partial credit. For questions 18–20, allow credit if the pupil has written the correct expression instead of the letter *a*, *b* or *c*.

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|------------------|----------------|
| (1) $100\pi$     | (11) 72        |
| (2) $8\sqrt{3}$  | (12) sometimes |
| (3) 7            | (13) sometimes |
| (4) 60           | (14) never     |
| (5) $64\sqrt{3}$ | (15) always    |
| (6) 5            | (16) always    |
| (7) $80\pi$      | (17) sometimes |
| (8) 3:2          | (18) <i>a</i>  |
| (9) $\pi ab$     | (19) <i>b</i>  |
| (10) 5           | (20) <i>c</i>  |