The University of the State of New York

313TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Wednesday, August 22, 1951 — 12 m. to 3 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in solid geometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1951 or number and length in minutes of lessons taken in the summer of 1951 under a tutor licensed in the subject and supervised by the principal of the school you last attended, (*d*) author of textbook used.

The minimum time requirement is four or five recitations a week for half a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1951 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

Answer two questions from part II.

21 Prove that if two planes are perpendicular to each other, a line drawn in one of them perpendicular to their intersection is perpendicular to the other. [10]

22 Two elements of a cylinder are *a* and *b*. A plane through *a* intersects a plane through *b* in line *c*. Prove that *c* is parallel to *a*. [10]

23 Prove that every section of a circular cone made by a plane parallel to its base is a circle. [10]

24 Given plane *P* and line *l*.

a Describe the locus of points

(1) *d* distance from *P* [3]

(2) *r* distance from *l* [3]

b What is the intersection of the two loci given in answer to *a* if

(1) *l* is perpendicular to *P* [2]

(2) *l* is in *P* and *d* is less than *r* [2]

[OVER]

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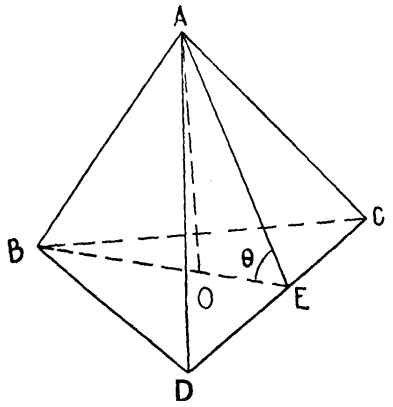
Part III

Answer three questions from part III.

25 Each of the piers used in building a power line is in the form of a frustum of a regular square pyramid whose base edges are 12 inches and 15 inches and whose altitude is 30 inches. Find, to the *nearest tenth of a cubic yard,* the amount of concrete necessary to build 80 such piers. [$V = \frac{1}{3}h\,(B + B' + \sqrt{BB'})$] [10]

26 On a sphere whose radius is 12, a spherical triangle has the same area as a zone whose altitude is $\frac{7}{2}$. If the angles of the triangle are in the ratio of 5:6:8, find each angle of the triangle. [10]

27 The base edge of a regular triangular pyramid is e and the slant height AE makes with the base an angle θ as shown in the diagram.



a If AO represents the altitude of the pyramid and V its volume, show that

(1) $OE = \dfrac{e\sqrt{3}}{6}$ [3]

(2) $V = \dfrac{e^3 \tan\theta}{24}$ [3]

b If $e = 2.76$ and $\theta = 52°$, find the volume to the *nearest tenth.* [4]

28 The entire amount of rain that falls on a flat roof 20 ft. by 30 ft. drains into a circular cistern 6 ft. in diameter. During a certain rainfall the water in the cistern rose 10 inches. Find, to the *nearest hundredth of an inch,* the amount of rain that fell. [The "amount of rain" is equal to the depth in inches of water that would be on the roof if all of it remained there.] [10]

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.

Directions (questions 1–11) — Write the answer to *each* question on the line at the right.

1 A lateral edge of a prism is 11 and its right section is a regular pentagon whose side is 3. Find the lateral area of the prism. 1.....

2 The altitude of a prism is 8 and the base is an equilateral triangle whose side is 6. Find the volume of the prism. [Answer may be left in radical form.] 2.....

3 The base edge of a regular square pyramid is 10 and the altitude is 12. Find the slant height of the pyramid. 3.....

4 The radii of the bases of a frustum of a right circular cone are 4 and 7 and the slant height is 4. Find the lateral area. [Answer may be left in terms of π.] 4.....

5 A rectangle 5 by 9 is revolved about the longer side as an axis. Find the *total* area of the cylinder thus formed. [Answer may be left in terms of π.] 5.....

6 Express the lateral area of a right circular cone in terms of its radius r and its slant height l. 6.....

7 The number of spherical degrees in the area of an equilateral spherical triangle is 36. Find the number of degrees in one angle of the triangle. 7.....

8 A point of light is 6 inches from a wall. A piece of cardboard whose area is 16 square inches is held parallel to the wall and 2 inches from it. Find the area of the shadow of the cardboard on the wall. 8.....

9 The volumes of two similar pyramids are in the ratio 1 : 8. If the altitude of the smaller is 1, find the altitude of the larger. 9.....

10 An angle of a spherical triangle is 71°. Find the number of degrees in the side opposite it in the polar triangle. 10.....

11 Find the radius of a sphere whose volume is 288π. 11.....

Directions (questions 12–16) — If the blank space in each statement is filled by one of the words, *always, sometimes* or *never,* the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

12 A plane passed through a line oblique to a plane is ... oblique to the plane. 12.....

13 Two obtuse angles whose sides are parallel are ... equal. 13.....

14 If two face angles of a trihedral angle are each less than 45°, the third face angle is ... acute. 14.....

15 If two spherical triangles on the same sphere have three angles of one triangle respectively equal to three angles of the other, the triangles are ... symmetrical. 15.....

16 An exterior angle of a spherical triangle is ... equal to the sum of the two nonadjacent interior angles. 16.....

Directions (questions 17–20) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a*, *b*, or *c*.

17 The number of planes that can be constructed perpendicular to each of two intersecting lines is (*a*)zero (*b*)one (*c*)more than one 17.....

18 The sum of the sides of a spherical triangle is always (*a*)more than 180° (*b*)equal to 360° (*c*)less than 360° 18.....

19 The number of spherical degrees in a lune whose angle is x degrees is (*a*)$\frac{1}{2}x$ (*b*)x (*c*)$2x$ 19.....

20 A sphere can be inscribed in any (*a*)parallelepiped (*b*)tetrahedron (*c*)cone 20.....