The University of the State of New York

307TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Tuesday, August 23, 1949 — 8.30 to 11.30 a. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in solid geometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1949 or number and length in minutes of lessons taken in the summer of 1949 under a tutor licensed in the subject and supervised by the principal of the school you last attended, (*d*) author of textbook used.

The minimum time requirement is four or five recitations a week for half a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1949 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

Answer two questions from part II.

21 Prove that if two angles not in the same plane have their sides respectively parallel and extending in the same direction from their vertices, they are equal. [10]

22 Line a is perpendicular to plane M at point P and b is any line in M through P. Prove that any line in M which is perpendicular to b is perpendicular to the plane determined by a and b. [10]

23 Prove that a spherical angle is measured by the arc of the great circle described from its vertex as a pole and included between its sides, extended if necessary. [10]

24 Given points A and B which are d distance apart and also point P which moves so that it is always at a given distance r from A and B.

a If $r > \frac{d}{2}$,

(1) Prove that P lies in a plane which is perpendicular to AB at its midpoint. [3]

(2) Prove that P lies on a circle whose center is the midpoint of AB. [4]

(3) Express the radius of this circle in terms of r and d. [1]

b What must be the relation between r and d if the locus of P is a point? If there is no such locus? [1, 1]

Part III

Answer three questions from part III.

25 A rectangular bar of lead whose volume is 22.8 cu. in. is melted and cast into a sphere. Allowing 6% for waste, find, to the *nearest tenth of an inch,* the radius of the sphere. [Use $\pi = 3.14$] [10]

26 The sides of a spherical triangle ABC are $x°$, $y°$, and $z°$.

a Show that the spherical excess of its polar triangle $A'B'C'$ is $360° - (x + y + z)°$. [4]

b Find in spherical degrees the area of triangle $A'B'C'$ if the sides of ABC are 82°, 86°, and 101°. [2]

c If $A'B'C'$ is drawn on a sphere whose radius is 6 inches, find, to the *nearest square inch,* the area of $A'B'C'$. [Use $\pi = \frac{22}{7}$] [4]

27 In trapezoid $ABCD$, $\angle B = 90°$, $\angle A = 60°$, $AB = 8$ inches, $DC = 5$ inches. The trapezoid is revolved through 360° about leg BC as an axis. Find the total area and the volume of the resulting solid. [Answers may be left in terms of π and radicals.] [7, 3] [The volume of a frustum of a circular cone is given by the formula $V = \frac{\pi h}{3}(r_1^2 + r_2^2 + r_1 r_2)$.]

28 The lateral area of a regular square pyramid, one of whose base edges is represented by e, is the area of its base.

a Find, to the *nearest degree,* the angle at which the slant height of the pyramid is inclined to the base. [6]

b Show that the volume of the pyramid is approximately $0.22e^3$. [4]

Fill in the following lines:

Name of pupil.....Name of school .....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.

Directions (questions 1–6) — In *each* of the following if the statement is *always* true, write *true* on the dotted line at the right, if *not always* true, write *false*.

1 Lines which are perpendicular to the same line at the same point are coplanar. 1.....

2 Two planes perpendicular to the same plane are parallel. 2.....

3 Two parallel line segments which are oblique to a plane are proportional to their projections on that plane. 3.....

4 If two lines are cut by three or more planes so that the corresponding segments are proportional, the planes are parallel. 4.....

5 Two trihedral angles are congruent if the three face angles of one are equal to the three face angles of the other and are arranged in the same order. 5.....

6 If a line is parallel to each of two intersecting planes, it is parallel to their line of intersection. 6.....

Directions (questions 7–10) — Indicate the correct answer to *each* question by writing the letter *a, b* or *c* on the dotted line at the right.

7 A sphere can be circumscribed about any (*a*) parallelepiped (*b*) right parallelepiped (*c*) rectangular parallelepiped 7.....

8 If three angles of a spherical quadrilateral are right angles, the fourth angle is (*a*) acute (*b*) right (*c*) obtuse 8.....

9 The sum of the sides of any convex spherical polygon is (*a*) less than 360° (*b*) greater than 180° (*c*) greater than 180° and less than 360° 9.....

10 If two face angles of a trihedral angle are 110° and 150°, the third face angle may be (*a*) 100° (*b*) 90° (*c*) 30° 10.....

11 Express the lateral area of a right circular cylinder in terms of its radius r and its altitude h. 11.....

12 The radius of a circular cone is $\frac{1}{3}$ its altitude h. Express the volume of the cone in terms of h. 12.....

13 The area of a zone is $\frac{1}{16}$ the area of the sphere on which it is drawn. Express the altitude of the zone in terms of the radius r of the sphere. 13.....

14 How many spherical degrees are there in a lune of 1°? 14.....

15 The lateral edge of an oblique prism is 6 and the area of a right section is 20. Find the volume of the prism. 15.....

16 Fruit juice is sold in cylindrical cans of two different sizes. The diameter and height of the large cans are twice the corresponding dimensions of the small cans. If a small can is priced at 15 cents, what should be the price of a large one? [Assume the cans are filled and disregard all other factors such as cost of handling, etc.] 16.....

17 The dimensions of a rectangular parallelepiped are in the ratio 1 : 2 : 3 and a diagonal of the parallelepiped is $\sqrt{56}$. Find the smallest dimension. 17.....

[OVER]

18 The altitude of a pyramid is 18 inches. How far from the vertex of the pyramid should a plane be passed parallel to the base so that the area of the section thus formed shall be $\frac{4}{9}$ the area of the base? 18.....

19 An edge of a regular octahedron is 5. Find its area. [Answer may be left in radical form.] 19.....

20 Given plane P and line r in P. How many lines are there which are 4 inches from P and 6 inches from r? 20.....