The University of the State of New York

298TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Wednesday, August 21, 1946 — 8.30 to 11.30 a. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish this part before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in solid geometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1946 or number and length in minutes of lessons taken in the summer of 1946 under a tutor licensed in the subject and supervised by the principal of the school you last attended, (*d*) author of textbook used.

The minimum time requirement is four or five recitations a week for half a school year. The summer school session will be considered the equivalent of one semester's work during the regular session (four or five recitations a week for half a school year).

For those pupils who have met the time requirement, the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1946 or an equivalent program of tutoring approved in advance by the Department is required.

Part II

Answer two questions from part II.

21 Prove that if each of two intersecting planes is perpendicular to a third plane, their intersection is also perpendicular to the given plane. [10]

22 Prove that the section of a prism made by a plane parallel to a lateral face is a parallelogram. [10]

23 Prove that if a pyramid is cut by a plane parallel to its base, the section is a polygon similar to the base. [10]

24 Each of the following statements is sometimes true and sometimes false. For *each* statement, give *one* condition under which the statement is *true*. [10]

a Three points determine a plane.

b If point A on a sphere is at a quadrant's distance from each of points B and C, also on the sphere, then A is the pole of arc BC.

c Two angles whose sides are respectively parallel are equal.

d A sphere can be circumscribed about a parallelepiped.

e A line perpendicular to each of two lines is perpendicular to the plane of the two lines.

Part III

Answer three questions from part III.

25 A watering trough is 10 feet long and 14 inches deep and the vertical cross section is a semicircle. Find, correct to the *nearest gallon*, the amount of water the trough holds. [Use $\pi = \frac{22}{7}$ and allow $7\frac{1}{2}$ gallons to a cubic foot.] [10]

26 A rectangular bar of metal 8 inches by 6 inches by 2 feet is to be melted and cast into spherical shot $\frac{1}{2}$ inch in diameter. If 8 cubic inches are allowed for waste, how many shot can be made? [Use $\pi = \frac{22}{7}$] [10]

27 The sides of a spherical triangle on a sphere of radius 8 are 80°, 100° and 120°.

a Find the area of the polar triangle. [Answer may be left in terms of π.] [6]

b A zone on the same sphere has an area equal to that of the polar triangle. Find the altitude of the zone. [4]

28 The slant height of the frustum of a regular square pyramid makes angle a with its projection on the larger base. If the edge of the larger base is m and the edge of the smaller base is n, show that the formula for the lateral area S of the frustum is

$$S = \frac{m^2 - n^2}{\cos a}$$ [10]

Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 The height of a cylinder of revolution is 5 and its lateral area is 60π. Find the radius of its base. 1.....

2 If the area of a lune is 70 spherical degrees, find the number of degrees in its angle. 2.....

3 The area of a lune whose angle is 45° is 18π sq. in. Find the radius of the sphere on which the lune is drawn. 3.....

4 Each face of a regular polyhedron is a square having an area of 5 sq. in. What is the total area of the polyhedron? 4.....

5 If two sides and the included angle of a spherical triangle are respectively 90°, 90° and 40°, how many degrees are there in the third side of the triangle? 5.....

6 Find the lateral area of a right circular cone whose altitude is 15 and the radius of whose base is 8. [Answer may be left in terms of π.] 6.....

7 The slant height of a regular square pyramid is 10 and the side of the base is 12. Find the volume of the pyramid. 7.....

8 An element of a cone makes an angle of 25° with the plane of the base. If the length of that element is 15 in., find, correct to the *nearest inch,* the altitude of the cone. 8.....

Directions (questions 9–12) — Indicate whether *each* statement is true or false by writing the word *true* or *false* on the line at the right.

9 Two planes perpendicular to the same plane are parallel to each other. 9.....

10 The edge of a dihedral angle is perpendicular to the plane of its plane angle. 10.....

11 A spherical triangle drawn on a given sphere is determined if the three sides are given. 11.....

12 The volume of a parallelepiped is equal to the product of its base and a lateral edge. 12.....

Directions (questions 13–20) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a, b, c,* or *d.*

13 This year a certain telephone system plans to install 2,100,000 miles of long distance circuits. The number of great circles of the earth to which this mileage is most nearly equal is (*a*) 4, (*b*) 8, (*c*) 80, (*d*) 160 13.....

14 That fractional part of the earth's surface which is included between meridians 15° E and 30° W is (*a*) $\frac{1}{8}$, (*b*) $\frac{1}{16}$, (*c*) $\frac{1}{24}$, (*d*) $\frac{1}{48}$ 14.....

15 If the altitude of a regular quadrangular pyramid is 12 and a base edge is 10, the total area of the pyramid is (*a*) 260, (*b*) 340, (*c*) 360, (*d*) 520 15.....

16 Which one of the following sets can be the angles of a spherical triangle? (*a*) 50°, 50°, 70°, (*b*) 30°, 70°, 80°, (*c*) 90°, 90°, 90°, (*d*) 20°, 90°, 130° 16.....

[OVER]

17 Which one of the following sets can be face angles of a trihedral angle? (*a*) 25°, 50°, 75°, (*b*) 50°, 60°, 70°, (*c*) 100°, 100°, 160°, (*d*) 50°, 90°, 160° 17.....

18 A plane passed parallel to the base of a cone bisects its altitude. The ratio of the volume of the original cone to that of the cone cut off by the plane is (*a*) 2 : 1, (*b*) 4 : 1, (*c*) 7 : 1, (*d*) 8 : 1 18.....

19 Two planes, *m* and *n*, intersect in line *AB*. Line *CD* lies in plane *m* and is parallel to plane *n*. Which of the following is true?

(*a*) *AB* and *CD* intersect.

(*b*) *AB* and *CD* are parallel.

(*c*) It is impossible to determine whether *AB* and *CD* are parallel or intersect. 19.....

20 *A* and *B* are two fixed points. The locus of point *C*, which moves so that angle *ACB* is always a right angle, is (*a*) a circle, (*b*) a sphere, (*c*) a plane parallel to *AB*, (*d*) a cylindrical surface 20.....