The University of the State of New York

289TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, August 19, 1943 — 8.30 to 11.30 a. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish this part before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in solid geometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1943 or number and length in minutes of lessons taken in the summer of 1943 under a tutor licensed in the subject and supervised by the principal of school you last attended, (*d*) author of textbook used.

The minimum time requirement is five recitations a week for half a school year. The summer school session will be considered the equivalent of one semester's work during the regular session or five recitations a week for half a school year.

For those pupils who have met the time requirement, the minimum passing mark is 65 credits; for all others 75 credits.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1943 or an equivalent program of tutoring approved in advance by the Department is required.

Answer five questions from parts II and III, including at least two questions from each part.

Part II

Answer at least two questions from part II.

21 Prove that if a line is perpendicular to a plane, every plane passed through the line is perpendicular to the given plane. [10]

22 Prove that if a pyramid is cut by a plane parallel to its base:

a The edges are divided proportionally [4]

b The section is a polygon similar to the base [6]

23 If a line r and a plane P are parallel, prove that any plane perpendicular to r is also perpendicular to plane P. [10]

*24 Show how the formula for the volume of a right circular cone can be obtained from the prismatoid formula $V = \frac{1}{6} h(B + B' + 4m)$ [10]

* This question is based on one of the optional topics in the syllabus.

Solid Geometry

Part III

Answer at least two questions from part III.

[Unless otherwise stated, answers may be left in terms of π and radicals.]

25 *a* A regular triangular prism has an altitude of 10 and a base edge of 6. Find the total area of the prism. [4]

b The angles of a spherical triangle are in the ratio 1 : 2 : 3, with the smallest angle equal to 40°. If the radius of the sphere is 7 inches, find the number of square inches in the area of the triangle. [Use $\pi = \frac{22}{7}$] [6]

26 A solid is in the form of a right circular cylinder surmounted by a frustum of a right circular cone, with the larger base of the frustum coinciding with a base of the cylinder. The cylinder has a radius of 2 feet and a height of 9 feet. The frustum is 6 feet high and the radius of its upper base is 1 foot. Find the volume of the solid. [The formula for the volume of a frustum is $V = \frac{1}{3}\,h(B_1 + B_2 + \sqrt{B_1B_2}]$ [10]

27 In a regular square pyramid the slant height is s and makes with the base an angle θ. Show that the formula for the volume of the pyramid is $V = \frac{4}{3}\,s^3 \sin\theta \cos^2\theta$. [10]

28 A rectangle and a line parallel to the longer side of the rectangle are in the same plane. The dimensions of the rectangle are 2 inches by 8 inches. The distance of the line from the nearer 8-inch side is 3 inches. If the rectangle is revolved through 360° about the line as an axis, find the total area of the solid generated. [10]

SOLID GEOMETRY

Fill in the following lines:

Name of school...Name of pupil...

Part I

Answer all questions in part I. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–4) — If the blank in each statement is replaced by one of the words *always, sometimes* or *never,* the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

1 The bases of a frustum of a pyramid are ... similar polygons. 1................

2 The projection of a straight line on a plane is ... a line. 2................

3 The section made by a plane passing through three points on a sphere is ... a great circle of the sphere. 3................

4 Tangents are drawn from an external point to a sphere. The locus of the points of tangency is ... a great circle of the sphere. 4................

Directions (questions 5–17) — Write the answer to *each* question on the line at the right.

5 How many spherical degrees are there in a lune if the angle of the lune is 10°? 5................

6 Find the volume of a sphere whose area is 144π. [Answer may be left in terms of π.] 6................

7 Find the lateral area of the frustum of a right circular cone, the radii of whose bases are 4 and 6 and whose slant height is 12. [Answer may be left in terms of π.] 7................

8 In an equilateral spherical triangle the sum of the three sides is 150°. How many degrees are there in *each* angle of the polar triangle? 8................

9 If both the altitude and the radius of a right circular cone are doubled, by what number is the volume of the cone multiplied? 9................

10 The diameter of the base and the slant height of a right circular cone are each 6 inches. Find the radius of the sphere inscribed in the cone. [Answer may be left in radical form.] 10................

11 How many square feet of canvas are there in a conical tent whose slant height is 10 feet and the diameter of whose base is 14 feet? [Use $\pi = \frac{22}{7}$ and assume that the tent does not have a canvas floor covering.] 11................

12 If $A'B'C'$ is the polar triangle of spherical triangle ABC, how many degrees are there in the great circle arc AC'? 12................

13 A cylindrical can having a radius of 3 inches is partly filled with water. A ball dropped into the can is completely covered by the water and causes the water to rise 2 inches. Find, correct to the *nearest cubic inch,* the volume of the ball. [Use $\pi = 3.14$] 13................

14 A wooden cube with an edge of 3 inches is painted blue. If the cube is divided into 27 one-inch cubes, how many of the small cubes will have just one blue face? 14................

15 Two face angles of a trihedral angle are equal. Of the three face angles two are 30° and 70°. How many degrees are there in the third angle? 15................

16 A smokestack on a ship is 21 feet long and a right section is always a circle of radius 3 feet. Find the number of square feet to be covered in painting the outside of the stack. [Use $\pi = \frac{22}{7}$] 16................

[OVER]

17 The base of a prism is a square 4 inches on a side. Each lateral edge is 8 inches and makes an angle of 30° with the base. Find the number of cubic inches in the volume of the prism. 17................

Directions (questions 18–20) — For *each* statement indicate on the line at the right whether you have been given *too little* information, *just enough* information or *more* information than is needed to justify the conclusion.

18 If a line is perpendicular to each of two lines, it is perpendicular to the plane of the two lines. 18................

19 If the altitude of a zone is 4 and the diameter of the sphere of which the zone is a part is 8, the area of the zone can be found. 19................

20 If in each face of a dihedral angle a line is drawn perpendicular to the edge of the angle, these lines form the plane angle of the dihedral angle. 20................