The University of the State of New York

285TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Friday, August 21, 1942 — 8.30 to 11.30 a. m., only

Instructions

Do not open this sheet until the signal is given.

Part I

This part is to be done first and the maximum time allowed for it is one and one half hours.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Parts II and III

Write at top of first page of answer paper to parts II and III (*a*) names of schools where you have studied, (*b*) number of weeks and recitations a week in solid geometry previous to entering summer high school, (*c*) number of recitations in this subject attended in summer high school of 1942, (*d*) author of textbook used.

The minimum time requirement is five recitations a week for half a school year. The summer school session will be considered the equivalent of one semester's work during the regular session or five recitations a week for half a school year.

For admission to this examination attendance on at least 30 recitations in this subject in a registered summer high school in 1942 is required.

See instructions for parts II and III on page 1.

Part II

Answer three questions from this part.

21 Prove that all the perpendiculars that can be drawn to a given line at a given point lie in the plane perpendicular to the line at the point. [10]

22 Prove that the sum of any three face angles of a polyhedral angle of four faces is greater than the fourth face angle. [10]

23 Prove that the sum of the angles of a spheric triangle is greater than 180° and less than 540°. [10]

24 P is the mid-point of the edge of AB of the tetrahedron $A–BCD$. Prove that the plane passing through P, C and D divides the tetrahedron into two equal triangular pyramids. [10]

25 AB, BC and CD are three line segments not in the same plane. Prove that the plane determined by the mid-points of the three lines is parallel to the lines AC and BD. [10]

Part III

Answer two questions from this part.

26 A spheric triangle on a sphere of radius 12 has the same area as a lune on a sphere of radius 4. The angles of the triangle are 40°, 75° and 95°. Find the number of degrees in the angle of the lune. [10]

27 *a* A drug store serves 5-cent and 10-cent orders of a certain soft drink in glasses 4 inches and $4\frac{3}{4}$ inches high, respectively. The glasses are in the form of frustums of similar cones of revolution. Would a customer get more by ordering one 10-cent glass or two 5-cent glasses? [Give work to justify your answer.] [5]

b A certain hot-water tank has the form of a right circular cylinder. The length of the tank is 5 feet and the radius is 8 inches. Find the number of gallons of water the tank will hold. [231 cu. in. = 1 gal.] [5]

28 The altitude of a regular quadrangular pyramid is h and the angle made by a lateral face with the base is m.

a Derive a formula for the lateral area of the pyramid in terms of h and m. [5]

b Using the formula derived in answer to *a*, find, correct to the *nearest integer*, the lateral area when $h = 7$ and $m = 50°$. [5]

Solid Geometry

Fill in the following lines:

Name of school...Name of pupil...

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 If we consider the earth as a sphere, what fractional part of its surface is included between the two meridians of longitude, 35°W. and 50°W.? 1................

2 The volume of a pyramid is 50 cubic inches and its base is a square 5 inches on a side. Find the altitude of the pyramid. 2................

3 A point on the plane bisecting a right dihedral angle is 10 inches from each face of the angle. Find the distance of the point from the edge of the angle. [Answer may be left in radical form.] 3................

4 The radii of the bases of two similar cylinders are 5 inches and 7 inches. If the lateral area of the smaller cylinder is 300 square inches, find the lateral area of the larger cylinder. 4................

5 The bases of the frustum of a regular pyramid are hexagons 3 inches and 5 inches on a side and the slant height of the frustum is 4 inches. Find the lateral area. 5................

6 Each element of a circular cone makes an angle of 60° with the base. Find the lateral area of the cone if the slant height is 8. [Answer may be left in terms of π.] 6................

7 The total surface of a cube is 24 square inches. Find the number of inches in a diagonal of the cube. [Answer may be left in radical form.] 7................

8 An angle of a spheric triangle contains a degrees. Express in terms of a the number of degrees in the side of the polar triangle that is opposite this angle. 8................

9 If the number of square inches in the surface of a sphere is the same as the number of cubic inches in its volume, what is the radius of the sphere? 9................

10 A man fits up the subbasement of his home as a neighborhood air-raid shelter. Its dimensions are 20 ft $\times$ 15 ft $\times$ 7 ft. If he estimates 60 cu. ft of space per person, how many persons will the shelter accommodate? 10................

11 If the slant height of a regular tetrahedron is 6 inches, find its altitude. [Answer may be left in radical form.] 11................

12 The total area of a right circular cylinder is 48 π. The radius of the base is 3. Find the height of the cylinder. 12................

13 A hole in the ground made by an exploding bomb is in the form of an inverted circular cone. The depth of the hole is 7 feet and the diameter of the top is 12 feet. Using $\pi = \frac{22}{7}$, find the number of cubic feet of earth it will take to fill the hole. 13................

14 The outer surface of the dome on top of a bank building is to be painted. The dome is in the form of a zone of one base and has a height of 10 feet. The radius of the sphere of which the zone would be a part is 28 feet. Find the number of square feet to be painted. [Use $\pi = \frac{22}{7}$] 14................

15 A line l oblique to plane P intersects P in point A. The number of lines perpendicular to l that can be drawn in plane P through A is (a) 0, (b) 1, (c) 2, (d) infinite. Which is correct, a, b, c or d? 15...............

16 The sides of a spheric triangle are 50°, 60° and 70° and the radius of the sphere is 7 inches. Using $\pi = \frac{22}{7}$, find the number of inches in the perimeter of the triangle. 16...............

Directions (questions 17–20) — Indicate whether each of the following statements is *always* true, *sometimes* true or *never* true by writing the word *always, sometimes* or *never* on the dotted line at the right.

17 Two lines perpendicular to the same line are parallel. 17...............

18 The projection of a line segment on a plane is longer than the line segment. 18...............

19 If two lines are parallel to a given plane, the plane of the lines is parallel to the given plane. 19...............

20 The locus of points in space equidistant from the four vertices of a rectangle is the line perpendicular to the plane of the rectangle at the point of intersection of the diagonals. 20...............