The University of the State of New York

Centennial of Regents Examinations

1865–1965

12B

REGENTS HIGH SCHOOL EXAMINATION

TWELFTH YEAR MATHEMATICS

12B (Solid Geometry)

Friday, June 18, 1965 — 1:15 to 4:15 p.m., only

The last page of the booklet is the answer sheet, which is perforated. Fold the last page along the perforation and then, slowly and carefully, tear off the answer sheet. Now fill in the heading of your answer sheet. When you have finished the heading, you may begin the examination immediately.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Write your answers in the spaces provided on the separate answer sheet.

1 Each lateral edge of a prism is 12 inches and makes an angle of 60° with the plane of the base. Express in radical form the number of inches in the altitude of the prism.

2 In two circular cylinders, A and B, the radius of A is 3 inches and its altitude is 4 inches; the radius of B is 4 inches and its altitude is 3 inches. What is the ratio of the volume of A to the volume of B?

3 The total areas of two similar pyramids, A and B, are in the ratio 4 : 1. Find the ratio of the volume of pyramid A to the volume of pyramid B.

4 The area of the base of a pyramid is 40 square inches. Find the number of square inches in the area of a section made by a plane that is parallel to the base and bisects the altitude.

5 If the number of cubic units in the volume of a sphere is equal to the number of square units in the area of the sphere, find the length of the radius of the sphere.

6 The altitude of a right circular cylinder is equal to the diameter of its base. If the volume of the cylinder is 128π cubic inches, find the number of inches in the length of the radius of the base.

7 On a sphere, a lune has a 20° angle and an area of 2π square units. Find the number of units in the radius of the sphere.

8 The slant height of a regular square pyramid is 13 inches and an edge of the base is 10 inches. Find the number of cubic inches in the volume of the pyramid.

9 The radius of a sphere is 20. What fractional part of the surface of the sphere is contained in a zone whose altitude is 8?

10 In rectangle $ABCD$, $AB = 6$ and $BC = 4$. Find in terms of π the *total* area of the cylinder of revolution obtained by rotating the rectangle about side AB as an axis.

11 A line oblique to a plane and its projection upon the plane determine a second plane. What is the number of degrees in the dihedral angle formed by the two planes?

12 The total area of a cone of revolution is $1\frac{1}{2}$ times the lateral area. If the slant height of the cone is 6 inches, find the number of inches in the radius of the base of the cone.

13 The base edges of a frustum of a regular square pyramid are 6 inches and 8 inches, respectively. If the slant height of the frustum is 5 inches, find the number of square inches in the lateral area of the frustum.

14 Three concurrent edges of a rectangular solid are proportional to 3, 4 and 5. If the length of a diagonal of the solid is $15\sqrt{2}$, find the length of the shortest edge.

[OVER]

15 The area of an equilateral spherical triangle is one-twelfth that of the sphere on which it is drawn. Find the number of degrees in one angle of the triangle.

16 The base of a prism is an equilateral triangle with side 4. The altitude of the prism is 10. Find its volume in radical form.

17 A triangle with sides 5, 12 and 13 is rotated about its shortest side as an axis. Find, in terms of π, the lateral area of the solid which is generated.

18 Find the sum of the number of faces on the five different regular polyhedrons.

Directions (19–26): Write on the separate answer sheet the *number* preceding the expression that best completes *each* statement or answers *each* question.

19 An edge of a cube is 8 inches. If each edge is increased 0.1 inch, which number most closely approximates the increase in the volume of the cube?
(1) 0.001 (3) 8
(2) 0.3 (4) 20

20 What is the locus of all lines through a point outside a plane and parallel to that plane?
(1) 1 line (3) 1 plane
(2) 2 lines (4) 2 planes

21 If two sides of a spherical triangle are 100° and 140°, then the third side may be
(1) 30° (3) 80°
(2) 40° (4) 120°

22 Two planes must be parallel if
(1) they are parallel to the same line
(2) they are perpendicular to the same line
(3) one contains a line parallel to the other
(4) one contains two lines parallel to the other

23 The number of points on a sphere equidistant from the vertices of a spherical triangle on that sphere is
(1) 1 (3) 3
(2) 2 (4) infinitely many

24 The projection of a square on a plane can *not* be
(1) a line segment (3) a parallelogram
(2) 2 parallel lines (4) a square

25 The perimeter of a spherical triangle is 300°. The spherical excess of its polar triangle is
(1) 60 (3) 180
(2) 120 (4) 240

26 The lateral faces of a regular pyramid cannot be equilateral triangles if the base is a
(1) square (3) hexagon
(2) pentagon (4) triangle

Directions (27–30): If the blank space in each statement below is replaced by the word *always*, *sometimes* (but not always), or *never*, the resulting statement will be true. Select the word that will correctly complete each statement and write this word in the space provided on the separate answer sheet.

27 If two planes are perpendicular, a line in one plane is...parallel to the other plane.

28 If two small circles of a sphere have the same pole, the planes of the circles are...parallel.

29 Two symmetric spherical triangles on the same sphere are...equal.

30 Point Q is the center of a small circle of sphere O. Point P is on the small circle. Plane M is tangent to the sphere at P. Line QP is...perpendicular to M.

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove *a* or *b* but not both: [10]

a Two planes perpendicular to the same line are parallel.

OR

b A spherical angle is measured by the arc of the great circle described from its vertex as a pole and included between its sides, produced if necessary.

32 Line l is perpendicular to plane P at point A.

a Describe fully the locus of points in space at a distance d from line l. [2]

b Describe fully the locus of points in space at a distance n from point A. [2]

c Describe fully the locus of points in space at a distance m from plane P. [2]

d If $n > d$, *name* the locus of points common to *a* and *b*. [2]

e [Write the number 1, 2, 3 or 4 on your answer paper after the letter *e*.] [2] If $d = n = m$, the number of points common to *a*, *b* and *c* is

(1) 1 (3) 0
(2) 2 (4) 4

33 Given triangular pyramid O–ABC. Point P is on edge OA, and PR and PS are perpendiculars drawn from P to the planes of the faces OBC and ABC, respectively. Prove that edge BC is perpendicular to the plane PRS. [10]

34 The sides of a spherical triangle are 60°, 70° and 80°. The area of its polar triangle is $\frac{15\pi}{2}$. A lune and a zone are drawn on the same sphere and the area of each is equal to the area of the polar triangle.

a Find the radius of the sphere. [6]

b Find the angle of the lune. [2]

c Find the altitude of the zone. [2]

35 A regular octahedron is inscribed in a sphere of radius r.

a Find the volume of the octahedron in terms of r. [5]

b Find the area of the octahedron in terms of r. [5]

36 The upper base of a prismatoid is a rectangle 8 inches by 6 inches and the lower base is a rectangle 14 inches by 8 inches. The distance between the bases is 10 inches and the longer sides of the upper and lower bases are parallel. Find the number of cubic inches in the volume of the prismatoid. [10]

*37 Answer *a* or *b* but *not* both:

a The coordinates of three points are A (0,3,0), B (0,0,4) and C (8,—5,2).

(1) Write an equation of the plane through A and B and parallel to the x-axis. [2]

(2) Write an equation of the plane through C and parallel to the xy-plane. [2]

(3) Write an equation of the sphere with its center at the origin and passing through point C. [2]

(4) If the points A, B and C are joined by line segments to form triangle ABC, find the length of the median of the triangle drawn to side AC. [4]

OR

b In right spherical triangle ABC, angle $C = 90°$, angle $A = 130°$ and side $b = 55°$. Find side c to the *nearest degree*. [10]

* This question is based on optional topics in the syllabus.

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ANSWER SHEET

Pupil..... Teacher.....

School.....

Name and author of textbook used.....

All of your answers to part I should be recorded on this answer sheet.

Part I

Answer all questions in this part.

1.....	11.....	21.....
2.....	12.....	22.....
3.....	13.....	23.....
4.....	14.....	24.....
5.....	15.....	25.....
6.....	16.....	26.....
7.....	17.....	27.....
8.....	18.....	28.....
9.....	19.....	29.....
10.....	20.....	30.....

Part I Score:.....

Rater's Initials:

.....

Tear Here

Your answers for part II should be placed on paper supplied by the school.

327

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FOR TEACHERS ONLY

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SCORING KEY

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Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 19–26, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) $6\sqrt{3}$

(2) $\frac{3}{4}$

(3) $\frac{8}{1}$

(4) 10

(5) 3

(6) 4

(7) 3

(8) 400

(9) $\frac{1}{8}$

(10) 80π

(11) 90

(12) 3

(13) 140

(14) 9

(15) 80

(16) $40\sqrt{3}$

(17) 156π

(18) 50

(19) 4

(20) 3

(21) 3

(22) 2

(23) 2

(24) 2

(25) 1

(26) 3

(27) sometimes

(28) always

(29) always

(30) never

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics*. Care should be exercised in making deductions as to whether the error is purely mechanical or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent depending on the relative importance of the principle in the solution of the problem.

(32) *a* A cylinder with l as axis and d as radius [2]
b A sphere with center A and radius n [2]
c Two planes parallel to P, one on either side, and at the distance m from P [2]
d two circles [2]
e 3 [2]

(34) *a* 3 [6]
b 75° [2]
c $\frac{5}{4}$ [2]

(35) *a* $\frac{4r^3}{3}$ [5]
b $4r^3\sqrt{3}$ [5]

(36) 780 [10]

(37) *a* (1) $\frac{y}{3} + \frac{z}{4} = 1$ [2]
(2) $z = 2$ [2]
(3) $x^2 + y^2 + z^2 = 93$ [2]
(4) $\sqrt{26}$ [4]
b 114° [10]