TWELFTH YEAR MATHEMATICS
12B (Solid Geometry)

Monday, June 18, 1962 — 1:15 to 4:15 p.m., only

Name of pupil...Name of school...

Name and author of textbook used...

Name of teacher...

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1 A face diagonal of a cube is $5\sqrt{2}$ inches. Express in cubic inches the volume of the cube.

1................

2 On a sphere of radius 10 inches, how many square inches will be covered by a zone of altitude 5 inches?

2................

3 Find the total surface area in square inches of a regular square pyramid, each of whose edges is $\sqrt{2}$ inches.

3................

4 Find the number of inches in the length of a tangent drawn to a sphere of radius 8 inches, if the tangent is drawn from a point 17 inches from the center of the sphere.

4................

5 Find the number of square inches in the area of the surface of the regular icosahedron whose edge is $2\sqrt{3}$ inches.

5................

6 The slant height of a regular tetrahedron is 15. Find the length of its altitude.

6................

7 A line segment 24 inches long is projected on a plane. If the line segment makes an angle of 60° with the plane, find in inches the length of the projection of the line segment on the plane.

7................

8 The volumes of two similar triangular prisms are in the ratio of 27:8. Express numerically the ratio of two corresponding altitudes.

8................

9 The perimeter of an equilateral spherical triangle is 300°. Find the number of degrees in the sum of the angles of its polar triangle.

9................

[OVER]

10 The area of a sphere is 60 square inches and the area of a lune on the sphere is 12 square inches. Find the number of degrees in the angle of the lune.

10....................

11 The diameter of the base of a cone of revolution is equal to its slant height. Express numerically the ratio of the total surface of the cone to its lateral surface.

11....................

12 A semicircle of radius 3 inches is rotated through 360° about its diameter as an axis. Find the number of cubic inches enclosed by the resulting surface.

12....................

13 A cylinder of altitude $2k$ is inscribed in a sphere of radius r. Write in radical form an expression for the radius of the cylinder in terms of k and r.

13....................

14 A triangle whose sides are 4, 4 and 6 is rotated through 360° about its longest side. What is the volume of the resulting solid?

14....................

15 A cube of edge e is inscribed in a sphere of radius r. Express e in terms of r.

15....................

16 A cone whose base has a radius of 1 inch and whose altitude is 6 inches is cut into 2 parts by a plane midway between the vertex and base, parallel to the base. What is the volume of the larger part?

16....................

Directions (17–25): Indicate the correct completion for *each* of the following by writing on the line at the right the *number* 1, 2, 3 or 4.

17 Two sides of a spherical triangle are 40° and 50°. The third side must have a value between

(1) 10° and 90°
(2) 180° and 540°
(3) 0° and 10°
(4) 90° and 270°

17......

18 AE and DF are skew lines. The total number of common perpendiculars that can be drawn between AE and DF is

(1) 1
(2) 2
(3) 3
(4) infinite

18......

19 The locus of points equidistant from the faces of a trihedral angle is

(1) 1 point
(2) 2 points
(3) a line
(4) a plane

19......

20 Line l is perpendicular to plane p. Line m is perpendicular to plane p. If line q intersects l and m,

(1) l and m are skew
(2) l intersects m
(3) q, l and m are coplanar
(4) q is perpendicular to p

20......

[2]

21 Point P is outside a sphere. The total number of planes through P and tangent to the sphere is

(1) 1
(2) 2

(3) infinite
(4) 0

21........

22 The locus of all points 5 inches from a given line in a given plane and also 4 inches from the plane is

(1) a pair of circles 3 inches from the plane
(2) 2 lines 3 inches apart
(3) a cylinder of radius 3 inches
(4) 4 lines parallel to the given plane

22........

23 The greatest possible number of edges of a face of any regular polyhedron is

(1) 5
(2) 6

(3) 3
(4) 4

23........

24 The locus of all points in space equidistant from two intersecting planes is

(1) one plane
(2) two planes

(3) one line
(4) a cylindrical surface

24........

25 Two face angles of a trihedral angle are 80° and 90°. The number of degrees in the third face angle may be

(1) 10°
(2) 170°

(3) 160°
(4) 360°

25........

Directions (26–30): If the blank space in each statement below is replaced by the word *always*, *sometimes* (but not always) or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

26 Two lines parallel to the same plane ... intersect each other.

26............

27 If two planes are perpendicular to the same plane, then the three planes ... have a point in common.

27............

28 The length of a diagonal of a cube is ... less than the length of a diagonal of a face of the cube.

28............

29 The section of a circular cone made by a plane cutting the elements of the cone is ... a circle.

29............

30 A plane section containing the axis of a right cylinder is ... a rectangle.

30............

[3]

[OVER]

Part II

Answer four questions from this part. Show all work unless otherwise directed.

31 Prove *either* a or b but *not* both: [10]

    a If two angles not in the same plane have their sides respectively parallel and extending in the same direction from their vertices, they are equal and their planes are parallel.

<div align="center">OR</div>

    b If a point on a sphere is at a quadrant's distance from each of two other points on the sphere, not the extremities of a diameter, it is the pole of the great circle passing through these points.

32 A tetrahedron is cut by a plane parallel to two of its opposite (nonintersecting) edges. Prove that the resulting plane section is a parallelogram.   [10]

33 Point P is at a distance f from plane H.

    a Describe fully the locus of points in space at a distance g from P.   [3]
    b If $g > f$, name the locus of points on H at a distance g from P.   [2]
    c Represent in terms of f and g the length of the locus named in part b.   [5]

34 The total area of a frustum of a regular square pyramid is twice its lateral area. If the base edges are 20 inches and 30 inches, respectively, find the

    a slant height of the frustum   [4]
    b altitude of the frustum   [2]
    c volume of the frustum   [4]

35 The area of a spherical triangle is 330 square inches. If the angles of the triangle are $100°$, $75°$ and $65°$, find to the *nearest inch* the radius of the sphere on which the triangle lies. [Use the approximation $\pi = \frac{22}{7}$.]   [10]

*36 Answer *either* a or b but *not* both:

    a A tetrahedron has vertices $A\,(1, 0, 0)$, $B\,(0, 0, 3)$, $C\,(0, 2, 0)$, $D\,(0, 0, 0)$.

        (1) Write an equation of the plane ABC.   [3]
        (2) Find the coordinates of E, the midpoint of AC.   [2]
        (3) Find the length of DE.   [2]
        (4) Write an equation of a sphere with center at the origin and radius equal to DE.   [3]

<div align="center">OR</div>

    b A right spherical triangle ABC has $a = 70°$, $b = 55°$ and $C = 90°$. Find A and c, each to the *nearest degree*.   [10]

    * This question is based on optional topics in the syllabus.

FOR TEACHERS ONLY

12 B

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use checkmarks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow 2 credits for each correct answer; allow no partial credit. For questions 17–25, allow credit if the pupil has written the correct answer instead of the number 1, 2, 3 or 4.

(1) 125

(2) 100π

(3) $2 + 2\sqrt{3}$ or 5.5

(4) 15

(5) $60\sqrt{3}$ or 103.8

(6) $10\sqrt{2}$ or 14.1

(7) 12

(8) 3:2

(9) 240

(10) 72

(11) 3:2

(12) 36π

(13) $\sqrt{r^2 - k^2}$

(14) 14π

(15) $\dfrac{2r}{\sqrt{3}}$

(16) $\dfrac{7\pi}{4}$

(17) 1

(18) 1

(19) 3

(20) 3

(21) 3

(22) 4

(23) 1

(24) 2

(25) 3

(26) sometimes

(27) sometimes

(28) never

(29) sometimes

(30) always

[OVER]

Part II

Please refer to the Department's pamphlet *Suggestions on the Rating of Regents Examination Papers in Mathematics.* Care should be exercised in making deductions as to whether the error is purely mechanical or due to a violation of some principle. A mechanical error generally should receive a deduction of 10 percent while an error due to a violation of some cardinal principle should receive a deduction ranging from 30 percent to 50 percent depending on the relative importance of the principle in the solution of the problem.

(33) *a* A spherical surface with center at P and radius g [3]

      b A circle   [2]

      c $2\pi\sqrt{g^2 - f^2}$ [5]

(34) *a* 13 inches   [4]

      b 12 inches   [2]

      c 7,600 cu. in.   [4]

(35) 18   [10]

(36) *a* (1) $6x + 3y + 2z = 6$ [3]

         (2) $(\tfrac{1}{2}, 1, 0)$ [2]

         (3) $\tfrac{1}{2}\sqrt{5}$ [2]

         (4) $4x^2 + 4y^2 + 4z^2 = 5$ [3]

      b $A = 73$

      c $= 79$ [10]