SOLID GEOMETRY

Friday, June 13, 1958—1:15 to 4:15 p.m., **only**

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed. Unless otherwise specified, answers may be left in terms of π or in radical form.

1. A rectangular parallelepiped whose dimensions are 6, 3 and 2 is inscribed in a sphere. Find the diameter of the sphere. 1..............

2. The angle of a lune is 50°. Find the number of spherical degrees in the area of the lune. 2..............

3. The angles of a spherical triangle are 50°, 70° and 80°. Find the number of degrees in the perimeter of its polar triangle. 3..............

4. A pyramid is 10 inches high. The area of its base is 150 square inches. Find the number of square inches in the area of the section of the pyramid cut by a plane parallel to the base and 6 inches from the vertex. 4..............

5. A line segment 10 inches long is inclined to a plane at an angle of 43°. Find, to the *nearest tenth,* the number of inches in the length of its projection on the plane. 5..............

6. The edges of the upper and lower bases of a frustum of a regular square pyramid are a and b, respectively. If the slant height is s, express the lateral area of the frustum in terms of a, b and s. 6..............

7. The radius of a sphere is 5. What is the area of a section of this sphere cut by a plane 3 units from the center? 7..............

8. Find the lateral area of a right circular cylinder whose altitude is 3.5 and whose radius is 1. [Use the approximation $\pi = 22/7$.] 8..............

9. A cone of revolution has a slant height of 10, and the radius of its base is 6. Find the volume of the cone. 9..............

10. Express the total area of a regular octahedron in terms of its edge e. 10..............

Directions (11-16): Indicate the correct completion for *each* of the following by writing on the line at the right the letter a, b or c.

11. The sum of the angles of a spherical quadrilateral is 420°. The ratio of the area of the quadrilateral to the area of the sphere on which it is drawn is (a) 1:12 (b) 1:6 (c) 1:3 11..

12. The face angles of a trihedral angle may be (*a*) 60°, 80°, 30° (*b*) 100°, 120°, 140° (*c*) 20°, 80°, 100° 12........

13. The locus of points equidistant from the three vertices of a triangle is a (*a*) point (*b*) line (*c*) plane 13........

14. Two spherical triangles on the same sphere have three angles of one equal to three angles of the other. The triangles are always (*a*) congruent (*b*) symmetric (*c*) equal in area 14........

15. A regular polyhedron may have (*a*) 4 vertices and 8 edges (*b*) 6 vertices and 12 edges (*c*) 8 vertices and 8 edges 15........

16. The areas of the bases of two similar solids are in the ratio of 1:4. The volumes of these solids are in the ratio (*a*) 1:16 (*b*) 1:8 (*c*) 1:64 16........

Directions (17-20): For *each* of the following, tell whether the statement is always true, sometimes true or never true by writing the word *always*, *sometimes* or *never* on the line at the right.

17. Through a given point, one and only one plane can be passed parallel to each of two given straight lines. 17...............

18. An exterior angle of a spherical triangle is equal to the sum of the two nonadjacent interior angles. 18...............

19. The lateral area of a prism is equal to the product of the perimeter of its base and a lateral edge. 19...............

20. A sphere can be circumscribed about any tetrahedron. 20...............

Part II

Answer three questions from this part.

21. Prove: If each of two intersecting planes is perpendicular to a third plane, their intersection is also perpendicular to that plane. [10]

22. *AB* is a straight line parallel to plane *MN*. Prove that any plane perpendicular to *AB* is perpendicular to plane *MN*. [10]

23. Prove: A spherical angle is measured by the arc of a great circle described from its vertex as a pole and included between its sides, produced if necessary. [10]

24. *a*. Given plane *P* of unlimited extent, describe fully the locus of points 5 inches from plane *P*. [2]
  b. Given line *m* of unlimited length, describe fully the locus of points 3 inches from line *m*. [2]
  c. Name the locus of points satisfying the conditions given in both *a* and *b* if line *m* is
    (1) parallel to plane *P* and 6 inches from *P* [2]
    (2) parallel to plane *P* and 8 inches from *P* [2]
    (3) perpendicular to plane *P* [2]

25. A sphere of radius r is inscribed in a cube. Show that the ratio of their volumes is equal to the ratio of their surfaces. [10]

Part III

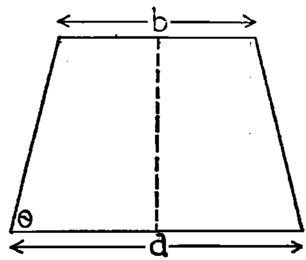
Answer two questions from this part. Show all work.

26. A spherical triangle whose angles are 110°, 60° and 70° is drawn on a sphere whose radius is 6 inches.
   a. Find in square inches the area of the spherical triangle. [Answer may be left in terms of π.] [4]
   b. A lune on the same sphere is equal in area to this triangle. Find the angle of the lune. [3]
   c. Find the altitude of a zone on the same sphere whose area is equal to the area of the triangle. [3]

27. A right circular cylinder of radius 6 inches is partly filled with water. When a sphere is completely immersed in the water in the cylinder, the surface of the water rises 4 inches. Find, to the *nearest tenth of an inch,* the radius of the sphere. [10]

28. An isosceles trapezoid is revolved through 180°, about the line joining the midpoints of its bases. The longer base of the trapezoid is a, the shorter base is b and a base angle of the trapezoid is θ. Show that the formula for the lateral area S of the solid generated is given by the formula

$$S = \frac{\pi(a^2 - b^2)}{4 \cos \theta}. \quad [10]$$



*29. Given the formula for the volume of a prismatoid

$$V = \frac{h}{6}(B + B' + 4m).$$

Considering each of the following solids as a special case of the prismatoid, show how to obtain the formulas for the volume of a (*a*) cylinder, (*b*) pyramid, (*c*) sphere. [3, 4, 3]

*This question is based upon an optional topic in the syllabus.

TWELFTH YEAR MATHEMATICS

12B (Solid Geometry)

Friday, June 13, 1958—1:15 to 4:15 p.m., only

Directions: The following questions are based upon the optional topics of the twelfth year syllabus. Question 30 may be substituted for any question in part II *only*. Question 31 may be substituted for any question in part III *only*.

Part II

30. *a*. Write an equation of the plane parallel to the xy-plane and 2 units below it. [2]
    b. Write an equation of the plane whose x-intercept is 2, whose z-intercept is —3 and which is parallel to the y-axis. [2]
    c. Write an equation of the sphere whose center is at the origin and which has a radius of 6. [2]
    d. Given the points $A(9, 2, -1)$ and $B(-3, 5, -5)$.
    (1) Find the coordinates of the midpoint of the line segment AB. [2]
    (2) Find the distance between A and B. [2]

Part III

31. In isosceles spherical triangle RST, $RT = ST$, $R = 70°$ and $T = 100°$. Find RT to the *nearest degree.* [10]

TWELFTH YEAR MATHEMATICS

12B (Solid Geometry)

Monday, June 15, 1959—1:15 to 4:15 p.m., only

Directions: The following questions are based upon the optional topics of the twelfth year syllabus. *Either one or both* may be substituted for *any one or two* of the questions on part II of the examination in solid geometry, including question 22, which concerns a theorem that is *not* among the required theorems of the twelfth year syllabus.

*29. In spherical triangle ABC, side $a = 37°$, side $b = 50°$ and angle $C = 90°$.
    a. Find side c to the *nearest degree.* [8]
    b. Using the given data, write an equation that could be used to find angle B. [2]

*30. A sphere has its center at the origin with point P $(2, 6, 3)$ on its surface. The sphere intersects the positive portions of the coordinate axes, x, y and z at points A, B and C, respectively.
    a. Write an equation of the plane through P parallel to the xz-plane. [2]
    b. Write an equation of the sphere. [4]
    c. Write an equation of the plane through A, B and C. [4]