SOLID GEOMETRY

Friday, June 15, 1956—1:15 to 4:15 p.m., **only**

Part I

Answer all questions in this part. Each correct answer will receive **2½** *credits. No partial credit will be allowed. Unless otherwise specified,* ***answers*** *may be left in terms of* π *or in radical form.*

1. A rectangular parallelepiped has the dimensions r inches, s inches and t inches. Express the number of inches in a diagonal in terms of r, s and t. 1.....

2. The radius of a cylinder of revolution is 4 and the altitude is 6. Find the lateral area of the cylinder. 2.....

3. A regular prism with a square base has an altitude of 16, and the perimeter of its base is 5. Find the volume of the prism. 3.....

4. A right circular cone has a base radius of 2 and a slant height of 7. Find the lateral area of the cone. 4.....

5. A regular pyramid with a square base has a base edge of 2 and an altitude of 12. Find the volume of the pyramid. 5.....

6. Find the lateral area of a frustum of a cone of revolution whose slant height is 5 and whose base radii are 3 and 4. 6.....

7. A regular pyramid has an altitude of 10 inches, and the area of a section made by a plane parallel to the base and 4 inches from the vertex is 4 square inches. Find the number of square inches in the area of the base of the pyramid. 7.....

8. The ratio of the lateral areas of two similar cylinders of revolution is 4:9. Find the ratio of the volume of the smaller cylinder to the volume of the larger. 8.....

9. A line 5.0 inches long is inclined at an angle of 28° to a plane. Find to the *nearest tenth of an inch* the length of the projection of the line on the plane. 9.....

10. A zone of a sphere of radius 1 has an altitude of ½. Find the area of the zone. 10.....

11. A side of an equilateral spherical triangle is 80°. Find the number of degrees in the sum of the angles of the polar triangle. 11.....

12. Two face angles of a trihedral angle are equal, and two of the face angles are 40° and 80°. How many degrees are there in the third face angle? 12.....

Directions (13-16): For *each* of the following, tell whether the statement is *always* true, *sometimes* true or *never* true by writing the word *always, sometimes* or *never* on the line at the right.

13. The area of the base of a pyramid is greater than the lateral area of the pyramid. 13.....

14. The locus of points at a given distance d from the intersection of two lines is a sphere whose radius is d and whose center is the point of intersection of the lines. 14.....

15. If a line is parallel to one plane and perpendicular to another plane, the two planes are perpendicular to each other. 15.....

16. If two lines are perpendicular, a plane containing one of the lines is perpendicular to a plane containing the other line. 16.....

Directions (17-20): Indicate the correct completion for *each* of the following by writing the letter *a*, *b* or *c* on the line at the right.

17. The number of planes which can be passed through one of two parallel lines and which will be parallel to the other line is (*a*) none (*b*) one (*c*) unlimited 17.....

18. If X represents the volume of a sphere of diameter 1 and if Y represents the volume of a rectangular solid whose dimensions are 1/3, 1 and 2, then X is (*a*) greater than Y (*b*) equal to Y (*c*) less than Y 18.....

19. The locus of points equidistant from the vertices of a tetrahedron is (*a*) a point (*b*) a line (*c*) a plane 19.....

20. If the angle of a lune is 40°, the ratio of the area of the lune to the area of the sphere on which it is drawn is (*a*) 1:18 (*b*) 1:9 (*c*) 2:9 20.....

Part II

Answer two questions from this part.

21. Prove: If a line is perpendicular to a plane, every plane passed through the line is perpendicular to the given plane. [10]

22. Prove: The sum of the angles of a spherical triangle is greater than 180° and less than 540°. [10]

23. Two cones of revolution having a common base are inscribed in a sphere. Prove that the sum of the volumes of the cones is to the volume of the sphere as the product of the altitudes of the cones is to twice the square of the radius of the sphere. [10]

24. Line m is parallel to a horizontal plane H and 6 units below H. Each locus listed in column *I* is described briefly *once* and *only once* in column *II*. List the numbers 1-5 on your answer paper, and after *each* number write the *letter* that indicates the description of that locus. [10]

Column I	*Column II*
(1) Locus of points at a distance d from H	(*a*) a line parallel to m
(2) Locus of points at a given distance 12 from m	(*b*) two lines parallel to m
(3) Locus of points satisfying both (1) and (2) when $d = 3$	(*c*) three lines parallel to m
(4) Locus of points satisfying both (1) and (2) when $d = 6$	(*d*) four lines parallel to m
(5) Locus of points satisfying both (1) and (2) when $d = 18$	(*e*) a plane parallel to H
	(*f*) two planes parallel to H
	(*g*) three planes parallel to H
	(*h*) a cylindrical surface
	(*i*) a spherical surface

Part III

Answer three questions from this part. Show all work.

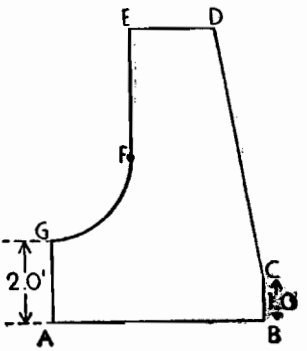
25. A spherical triangle on a sphere of radius 9.0 inches has an area of 198 square inches. Two of its angles are 115° and 135°. Find the third angle. [Use $\pi = 22/7$.] [10]

26. A landscape architect has designed an ornamental pool for a public park. The pool is to be in the form of a frustum of a regular square pyramid whose upper base edge is 96 feet, lower base edge 72 feet and altitude 5.0 feet. If the architect estimates the construction cost at 95 cents for each square foot of the lateral surface and 80 cents for each square foot of the bottom surface of the pool, find to the *nearest hundred dollars* an estimate of the cost of constructing the pool. [10]

27. *a.* Given a cone of revolution whose altitude is equal to half its slant height. Show that the slant height l may be expressed in terms of the volume V by the formula $l = \sqrt[3]{\frac{8V}{\pi}}$. [4]

b. Using the formula given in part *a*, find to the *nearest tenth of an inch* the slant height of such a cone whose volume is 767 cubic inches. [Use $\pi = 3.14$.] [6]

28. The drawing at the right represents a cross section of a concrete wall. ED is parallel to AB; the angles at A, B and E are right angles; FG is a quadrant of a circle. $AB = 5.0$ feet, $BC = 1.0$ feet, $DE = 2.0$ feet, $EF = 3.0$ feet, $AG = 2.0$ feet and the radius of the quadrant FG is 2.0 feet. If the wall is 100 feet long, find to the *nearest cubic yard* the amount of concrete necessary to construct the wall. [Use $\pi = 3.14$ and make no allowance for waste.] [10]



The University of the State of New York

327TH HIGH SCHOOL EXAMINATION

TWELFTH YEAR MATHEMATICS

12B (Solid Geometry)

Friday, June 15, 1956 — 1:15 to 4:15 p.m., only

Note to teacher: These questions may be used in conjunction with the regular Regents examination in solid geometry by those pupils who have followed the outline in the twelfth year syllabus. A copy of this sheet should be distributed to each pupil qualified, together with a copy of the regular examination paper in solid geometry. If sufficient copies of this sheet are not available, these questions may be written on the blackboard.

Part III

Directions: The following questions, 29 and 30, are based on optional topics of the twelfth year syllabus. *Either* 29 *or* 30, but *not both,* may be substituted for *any one* of the questions on part III of the examination in solid geometry.

29 Given spherical triangle ABC in which $A = 65°$, $B = 58°$ and $C = 90°$.

a Write an equation which expresses a trigonometric function of side c in terms of trigonometric functions of 65° and 58°. [3]

b Write an equation which expresses a trigonometric function of side a in terms of trigonometric functions of 65° and 58°. [3]

c Find side a to the *nearest degree.* [4]

30 The vertices of the base of a regular pyramid V–$ABCD$ are $A(-3, 0, 0)$, $B(0, 3, 0)$, $C(3, 0, 0)$ and $D(0, -3, 0)$. The altitude of the pyramid is 4 and vertex V is above the base.

a Find the coordinates of V. [2]

b Write an equation of the plane passing through V parallel to the base. [2]

c Write an equation of the plane containing the points A, B and V. [2]

d Find the coordinates of the midpoint M of the lateral edge AV. [2]

e Find the length of line segment MC. [Answer may be left in radical form.] [2]

FOR TEACHERS ONLY

SG

INSTRUCTIONS FOR RATING
SOLID GEOMETRY

Friday, June 15, 1956 — 1:15 to 4:15 p.m., only

Use only *red* ink or pencil in rating Regents papers. Do not attempt to *correct* the pupil's work by making insertions or changes of any kind. Use check marks to indicate pupil errors.

Unless otherwise specified, mathematically correct variations in the answers will be allowed. Units need not be given when the wording of the questions allows such omissions.

Part I

Allow $2\frac{1}{2}$ credits for each correct answer; allow no partial credit. For questions 17–20, allow credit if the pupil has written the correct answer instead of the letter *a*, *b* or *c*.

(1) $\sqrt{r^2 + s^2 + t^2}$

(2) 48π

(3) 25

(4) 14π

(5) 16

(6) 35π

(7) 25

(8) 8:27

(9) 4.4

(10) π

(11) 300

(12) 80

(13) never

(14) always

(15) always

(16) sometimes

(17) *c*

(18) *c*

(19) *a*

(20) *b*