SOLID GEOMETRY

Thursday, June 23, 1955—9.15 a.m. to 12.15 p.m., only

Part I

Answer all questions in this part. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed.

1. A cube has a volume of 64. Find its total area. 1...............

2. A rectangular parallelepiped has the dimensions 4, 6 and 12. Find a diagonal. 2...............

3. The perimeter of a right section of a prism is 20 and a lateral edge is 15. Find the lateral area of the prism. 3...............

4. A cone of revolution has an altitude of 10 inches, and the area of its base is 25 square inches. Find the number of square inches in the area of a section made by a plane parallel to the base and 4 inches from the vertex. 4...............

5. A regular pyramid with a square base has a base edge of 2 and a slant height of 6. Find the lateral area of the pyramid. 5...............

6. A lawn roller is in the form of a right circular cylinder whose altitude is 3 feet and the radius of whose base is 1 foot. If it is rolled through 7 complete revolutions, find the number of square feet it has covered. [Use $\pi = 22/7$.] 6...............

7. The radius of the base of a circular cylinder is three times the altitude h of the cylinder. Express the volume of the cylinder in terms of π and h. 7...............

8. Find the lateral area of a frustum of a regular triangular pyramid if the edges of the upper and lower bases are 3 and 4 respectively, and if the slant height is 5. 8...............

9. The ratio of the volumes of two similar cones of revolution is 8:27. Find the ratio of the altitude of the smaller cone to the altitude of the larger. 9...............

10. A line 10 inches long is inclined at an angle of 19° to a plane. Find, to the *nearest tenth of an inch*, the length of the projection of the line on the plane. 10...............

11. The area of a great circle of a sphere is 10. Find the area of the sphere. 11...............

12. The angles of a spherical triangle are 60°, 100° and 140°. Find the number of spherical degrees in the area of this triangle. 12...............

Directions (13-16): Indicate whether *each* statement is true or false by writing the word *true* or *false* on the line at the right.

13. The locus of points at a given distance m from a given line b is a circular cylindrical surface whose axis is b and whose radius is m. 13..............

14. The northern hemisphere of the earth is divided by the parallel of latitude at 45° N into two zones which are equal in area. 14..............

15. If a plane P is passed through a line c and through the projection of c on a plane Q, then plane P is perpendicular to plane Q. 15..............

16. If one side of a spherical triangle contains 60°, the angle opposite this side in the polar triangle contains 120°. 16..............

Directions (17-20): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b* or *c*.

17. The face angles of a trihedral angle may be (*a*) 55°, 75°, 125° (*b*) 100°, 125°, 150° (*c*) 75°, 100°, 125° 17..............

18. If the angle of a lune is equal to the spheric excess of a triangle on the same sphere, the area of the lune is (*a*) greater than the area of the triangle (*b*) equal to the area of the triangle (*c*) less than the area of the triangle 18..............

19. The volume of the sphere which is inscribed in a cube whose edge is 6 is (*a*) 288π (*b*) 45π (*c*) 36π 19..............

20. The distance between two points A and B is d. The locus of points at a distance d from both A and B is (*a*) a straight line (*b*) a circle (*c*) a plane 20..............

Part II

Answer two questions from this part.

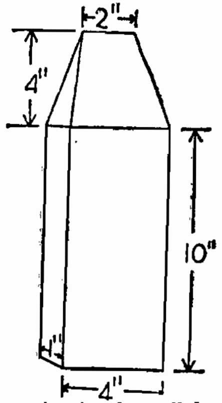
21. Prove: If a point on a sphere is at a quadrant's distance from each of two other points on the sphere, not the extremities of a diameter, it is the pole of the great circle passing through these points. [10]

22. Given a triangular pyramid $V\text{-}ABC$, with VC perpendicular to the plane ABC. If R and S are the midpoints of VA and AC respectively and T is a point on BC, prove that the plane RST is perpendicular to the plane ABC. [10]

23. *a*. Prove: Any point in the plane which is perpendicular to a given line segment at its midpoint is equidistant from the end points of the segment. [8]

b. State the converse of the theorem given in *a*. [2]

*24. A machine part has the dimensions shown on the figure; the upper portion of the machine part is a wedge whose altitude is 4 inches; the lower portion is a rectangular prism. If the machine part is made of an alloy which weighs 432 pounds per cubic foot, find its weight to the *nearest pound.* [A wedge is a special case of a prismatoid; $V = \frac{h}{6}(B + B' + 4M)$.] [10]



* *This question is based upon one of the optional topics in the syllabus. It may be used as one of the questions in part II or may be substituted for any one of the questions in part III.*

Part III

Answer three questions from this part. Show all work.

25. A bar of metal containing 144 cubic inches is melted, and the metal is recast into a solid circular cone whose altitude is one half the radius of its base. Find, to the *nearest tenth of an inch,* the radius of the base of the cone. [Use $\pi = 3.14$.] [10]

26. The area of an equilateral triangle on a sphere whose radius is 3.6 inches is 2π square inches. Find, to the *nearest degree,* each angle of the triangle. [10]

27. According to ancient manuscripts, each Egyptian pyramid was a regular pyramid with a square base. The altitude of each pyramid was designed to be equal to the radius of a circle whose circumference equalled the perimeter of the base of the pyramid.

a. Using $\pi = 22/7$, show that the altitude h of such a pyramid may be expressed in terms of an edge b of the base by the formula $h = \frac{7b}{11}$. [5]

b. Find, to the *nearest cubic inch,* the volume of a model of an Egyptian pyramid having a base edge of 10 inches. [5]

28. In the figure at the right, AB is a quarter circle with center at O and radius of 3; the line segment BC is parallel to AO and has a length of 3; and the angle BCD is 30°, the point D being on AO. Find the area of the surface generated by revolving $ABCD$ through 360° about AO as an axis. [Answer may be left in terms of π.] [10]