SOLID GEOMETRY

Thursday, June 24, 1954—9.15 a. m. to 12.15 p. m., only

Part I

Answer all questions in part I. Each correct answer will receive **2** *credits. No partial credit will be allowed.*

1. The area of the base of a pyramid is 33 and its altitude is 8. Find the volume of the pyramid. 1.....

2. Find the number of square inches in the lateral area of a regular square prism whose base edge is 15 inches and whose altitude is 10 inches. 2.....

3. The radius of a right circular cylinder is r and its altitude is $3r$. Express the total area of the cylinder in terms of r. [Answer may be left in terms of π.] 3.....

4. The areas of two similar metal cones are in the ratio 4:9. If the cones are made of the same material and the weight of the smaller is 16 ounces, find the weight of the larger in ounces. 4.....

5. The area of the base of a pyramid is 27 and the altitude of the pyramid is 9. If the area of a section parallel to the base is 12, find its distance from the vertex. 5.....

6. The edges of a rectangular parallelepiped are 3, 5 and $\sqrt{15}$. Find a diagonal. 6.....

7. The lateral area of a frustum of a right circular cone is 48π. If the slant height is 6 and the radius is 5, find the radius of the other base. 7.....

8. If the number of square feet in the area of a sphere is equal to the number of cubic feet in its volume, find the length in feet of its radius. 8.....

9. The area of a zone is one fourth the area of the sphere on which it is drawn. Express in terms of the radius the altitude of the zone. 9.....

10. The area of a lune is 30 spherical degrees. Find the number of degrees in the angle of the lune. 10.....

11. Find the area of a regular octahedron whose edge is 2. [Answer may be left in radical form.] 11.....

12. A line 8.5 inches long is inclined to a plane at an angle of 58°. Find to the *nearest tenth of an inch* the length of its projection on the plane. 12.....

Directions (13-15): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a*, *b* or *c*.

13. The number of planes that can be passed perpendicular to a given plane through a line oblique to the plane is (*a*)zero (*b*)one (*c*)more than one 13.....

14. The locus of points a given distance from a given plane and equidistant from the vertices of a triangle in the given plane is (*a*)a line (*b*)a point (*c*)two points **14**.....

15. The locus of points equidistant from two parallel planes and equidistant from the end points of a line segment oblique to these planes is (*a*)two planes (*b*)two lines (*c*)one line 15.....

Directions (16-20): For each of the following, tell whether the statement is always true, sometimes true or never true, by writing one of the words *always, sometimes* or *never* on the line at the right.

16. The intersection of a plane and a curved surface is a curved line. 16.....

17. If the three face angles of one trihedral angle are equal respectively to the three face angles of another trihedral angle, the trihedral angles are congruent. 17.....

18. If two face angles of a trihedral angle are each 135°, the third face angle is acute. 18.....

19. If two parallel line segments terminated by two planes are equal, the two planes are parallel. 19.....

20. A line parallel to one of two intersecting planes is parallel to the other. 20.....

Part II

Answer two questions from part II.

21. Prove: If two planes are perpendicular to each other, a line drawn in one of them perpendicular to their intersection is perpendicular to the other plane. [10]

22. Prove: A spherical angle is measured by the arc of a great circle described from its vertex as a pole and included between its sides, extended if necessary. [10]

23. *O–ABCD* is a square pyramid. *X* and *Y* are the mid-points of *OA* and *OB*. A plane through *X* and *Y* intersects *OC* in *R* and *OD* in S. Prove that *RS* is parallel to *CD*. [10]

24. Planes *M* and *N* intersect in line *x*, and *P* is a point on *x*.

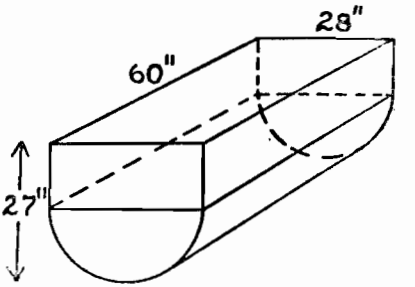
a. Describe *fully* the locus of points
(1) equally distant from *M* and *N* [3]
(2) a given distance *d* from *x* [3]
(3) a given distance *r* from *P* [2]

b. Indicate the correct completion for *each* of the following statements by writing the letter *a, b* or *c* after the proper number on your answer paper:
(4) The locus of points common to (1) and (2) above consists of (*a*)one line (*b*)two parallel lines (*c*) four parallel lines [1]
(5) The locus of points common to (1) and (3) consists of (*a*)two small circles (*b*)one great circle (*c*)two great circles [1]

Part III

Answer three questions from part III. All work, including computation, should be shown.

25. The upper part of a spray tank is a rectangular parallelepiped and the lower part is a half cylinder whose rectangular surface coincides with the base of the parallelepiped. The tank is 28 inches wide, 27 inches deep and 60 inches long, as shown in the drawing. Find to the *nearest gallon* the capacity of the tank. [Use $\pi = \frac{22}{7}$, 1 gallon = 231 cubic inches.] [10]



26. In spherical triangle ABC, angle B is 20° larger than angle A, and angle C is 30° larger than angle B. If the area of the triangle is 176 square inches and the radius of the sphere is 12 inches,

a. find the angles of the triangle [Use $\pi = \frac{22}{7}$.] [6]

b. find to the *nearest inch* the perimeter of the palor triangle [4]

27. A coal bin is in the form of a frustum of a regular square pyramid. The upper base edge is 16 feet, the lower base edge is 4 feet and the height is 12 feet. Find to the *nearest ton* the number of tons of coal in the bin when it is filled to a depth of 8 feet. [Allow 36 cubic feet for each ton.] [$V = \frac{1}{3}h\ (B + B' + \sqrt{BB'})$] [10]

28. Right triangle ABC with the right angle at C is revolved through 360° about BC as an axis.

a. If AC is represented by r, angle BAC by θ, and the volume of the resulting solid of revolution by V, show that

$$r = \sqrt[3]{\frac{3V}{\pi \tan \theta}}$$ [5]

b. If $V = 264$, $\theta = 54°$ and $\pi = 3.14$, find the value of r to the *nearest tenth.* [5]

TWELFTH YEAR MATHEMATICS
(Solid Geometry)

Note to teacher: This question may be used in conjunction with the regular Regents examination in solid geometry by those pupils who have followed the outline in the twelfth year syllabus. A copy of this sheet should be distributed to each pupil qualified, together with a copy of the regular examination paper in solid geometry. If sufficient copies of this sheet are not available, these questions may be written on the blackboard.

Part III

Directions: The following question is based on optional topics of the twelfth year syllabus and may be used in place of *any one* of the questions on part III of the examination in solid geometry.

29. Answer *either* *a* or *b*:

a. Given spherical triangle ABC in which angle $C = 90°$, angle $B = 70° 10'$ and side $c = 79° 40'$. Find angle A to the *nearest ten minutes.* [10]

b. Given the points $A(3, 0, 0)$, $B(0, 3, 0)$ and $C(0, 0, 3)$. Write the equation of

(1) the plane through the points A, B and C [3]

(2) the sphere whose center is at the origin and which passes through the points A, B and C [3]

(3) the planes tangent to the sphere described in (2) at the points where the z-axis intersects the sphere [4]