

The University of the State of New York

318TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, June 18, 1953 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in solid geometry, (c) author of textbook used.

The minimum time requirement is four or five recitations a week for half a school year.

Part II

Answer two questions from part II.

21 Prove that in two polar triangles, each angle of one has the same measure as the supplement of the side lying opposite to it in the other. [10]

22 Prove that if each of two intersecting planes is perpendicular to a third plane, their intersection is also perpendicular to that plane. [10]

23 Point P is $\frac{r}{2}$ units above plane m . Describe the locus of points

a at a given distance r from P [2]

b at a given distance d from m [2]

c that satisfy the conditions given in both a and b if

(1) $d = \frac{r}{2}$ [3]

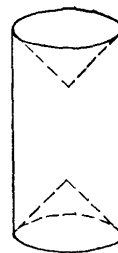
(2) d is less than $\frac{r}{2}$ [3]

24 In pyramid $O-ABC$, D and E are points in OB and OC respectively, such that $OD:DB = OE:EC$. A plane through DE cuts AB at G and AC at F . Prove that $AG:GB = AF:FC$. [10]

Part III

Answer three questions from part III.

25 The accompanying figure represents a solid formed by removing two right circular cones from a right circular cylinder. The altitude of the cylinder is 9.0 inches and its diameter is 5.6 inches. The altitude of each cone is 2.4 inches. Find to the nearest cubic inch the volume of the solid. [10]



[1]

[OVER]

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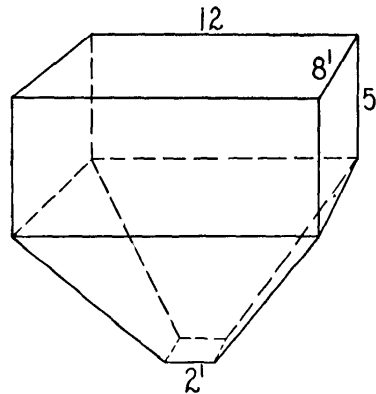
26 Each base angle of an isosceles spherical triangle is 75° . The equal sides produced form a lune whose area is four times the area of the triangle. Find the angle of the lune. [10]

27 a A lateral edge e of a regular square pyramid makes an angle θ with the base edge. Express the lateral area L of the pyramid in terms of e and θ . [6]

b Using the formula found in a, find L to the nearest square inch when e is 13.4 inches and θ is 55° . [4]

28 A regular prism with a square base is inscribed in a sphere. If the altitude of the prism is 8 and the diameter of the sphere is 12, show that the ratio of the volume of the prism to the volume of the sphere is $\frac{10}{9\pi}$. [10]

*29 The accompanying figure represents a bin for sand. The upper part is a rectangular solid whose altitude is 5 feet. The lower part is a prismaticoid whose altitude is 6 feet. The common base is a rectangle 12 feet long and 8 feet wide. The lower base of the prismaticoid is a square 2 feet on a side. Find to the nearest cubic yard the number of cubic yards needed to fill the bin. $[V = \frac{h}{6} (B + B' + 4m)]$ [10]



* This question is based upon one of the optional topics in the syllabus.

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive $2\frac{1}{2}$ credits. No partial credit will be allowed.

1 Find the lateral area of a prism whose lateral edge is 7 inches and whose right section is a regular hexagon 4 inches on a side. 1.....

2 If an edge of a regular tetrahedron is 6, find the total surface of the tetrahedron. [Answer may be left in radical form.] 2.....

3 The altitudes of two zones drawn on a given sphere are 2 and 5. Find the ratio of the areas of the zones. 3.....

4 The radii of the bases of a frustum of a right circular cone are 2 and 5 and the slant height is 7. Find the lateral area. [Answer may be left in terms of π .] 4.....

5 The area of a lune is 110 spherical degrees. Find the angle of the lune. 5.....

6 Find the radius of a sphere whose volume is $\frac{32}{3} \pi$. 6.....

7 The altitude of a pyramid is 9 inches and its base is 180 square inches. Find the area of the section made by a plane parallel to the base and 6 inches from it. 7.....

8 A square whose side is s is revolved through 360° about one side as an axis. Find the total area of the solid thus formed. [Answer may be left in terms of π .] 8.....

9 A line segment 5 inches long is inclined at an angle of 52° to a plane. Find to the *nearest inch* the length of its projection on the plane. 9.....

10 A paper box manufacturer doubles the dimensions of a rectangular box. By what number is the volume of the box multiplied? 10.....

11 Find the spherical excess of a spherical polygon with angles of 80° , 120° , 40° and 130° . 11.....

12 A zone whose altitude is 4 is drawn on a sphere whose area is 400π . Find the area of the zone. [Answer may be left in terms of π .] 12.....

13 The base edge of a regular square pyramid is 6 and its slant height is 5. Find its volume. 13.....

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Directions (14-17) — If the blank in each of the following statements is filled by one of the words *always*, *sometimes* or *never*, the resulting statement will be true. Select the word that correctly completes *each* statement and write this word on the line at the right.

14 Given plane P and line l not in P . It is ... possible to draw a line in P parallel to l . 14.....

15 It is ... possible to have a spherical triangle whose angles are 40° , 65° and 75° . 15.....

16 The projection of a triangle on a plane is ... a straight line segment. 16.....

17 If two planes are perpendicular to each other, a line parallel to one of the planes is ... perpendicular to the other. 17.....

Directions (18-20) — Indicate the correct completion for *each* of the following by writing on the line at the right the letter a , b or c .

18 Two spherical triangles on the same sphere that have three angles of one equal to the three angles of the other are always (a)congruent (b)symmetric (c)equal in area 18.....

19 The locus of points equidistant from the three vertices of a triangle is a (a)point (b)line (c)plane 19.....

20 The sides of a spherical triangle may be (a) 30° , 40° , 70° (b) 110° , 120° , 130° (c) 30° , 70° , 85° 20.....