The University of the State of New York

315TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, June 19, 1952—9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry, (*c*) author of textbook used.

The minimum time requirement is four or five recitations a week for half a school year.

Part II

Answer two questions from part II.

21 Prove that two lines perpendicular to the same plane are parallel. [10]

22 Prove that if the first of two spherical triangles is the polar triangle of a second, then the second is the polar triangle of the first. [10]

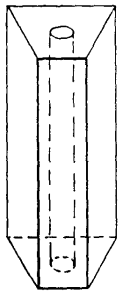
23 Given O and O', the intersections of the diagonals of the upper and lower bases of an oblique parallelepiped. Prove that OO' is parallel to a lateral edge of the parallelepiped. [10]

24 Prove that a plane perpendicular to one of two parallel planes is perpendicular to the other also. [10]

Part III

Answer three questions from part III.

25 The accompanying figure represents a metal casting in the form of a right prism through which a cylindrical hole has been bored. The base of the prism is an isosceles trapezoid whose parallel sides are 18 in. and 8 in. and whose legs are each 13 in. The height of the casting is 3 ft. and the diameter of the hole is 4 in. Find to the *nearest cubic foot* the volume of the casting. [10]



26 A lune having an area of 462 is drawn on a sphere whose radius is 21.

a Find the angle of the lune. [Use $\pi = \frac{22}{7}$] [4]

b An equilateral triangle on the same sphere has the same area as the lune. Find one angle of the triangle. [5]

c Find a side of the polar triangle. [1]

27 The lateral area of a right circular cone is 66 and its radius is 3. Find to the *nearest integer* the volume of the cone. [Use $\pi = \frac{22}{7}$] [10]

28 The base edge of a regular triangular pyramid is e and the slant height makes an angle θ with the base.

a Express the lateral area in terms of e and θ. [5]

b If $e = 2.97$ and $\theta = 57°$, find the lateral area to the *nearest tenth.* [5]

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 The area of a zone on a sphere of radius 6 is 24π. Find the altitude of the zone. 1.....

2 The radii of the bases of a frustum of a right circular cone are 3 and 6 and the slant height is 4. Find the lateral area of the frustum. [Answer may be left in terms of π.] 2.....

3 The dimensions of a rectangular parallelepiped are 3 in., 4 in. and 8 in. Find the length of a diagonal of the parallelepiped. [Answer may be left in radical form.] 3.....

4 The lateral area of a regular square prism is 84. If a lateral edge is 6, find one side of the base. 4.....

5 A line 12 in. long is inclined to a plane at an angle of 55°. Find, to the *nearest tenth of an inch,* the length of its projection on the plane. 5.....

6 Find the altitude of a pyramid whose volume is 60 cubic inches and whose base is 20 square inches. 6.....

7 The diameter d of the base of a right circular cylinder is equal to the altitude. Express its lateral area in terms of d. 7.....

8 The area of the base of a pyramid is 18 and its altitude is 6. The area of a section of this pyramid formed by a plane parallel to the base is 2. Find the distance from the vertex to this plane. 8.....

9 The radius of a sphere is 6 inches. Find the area in square inches of a spherical triangle on this sphere whose angles are 65°, 75° and 80°. [Answer may be left in terms of π.] 9.....

10 The volume of one cube is 64 times that of another. If the area of the smaller cube is s, express in terms of s the area of the larger cube. 10.....

11 Find the volume of a sphere whose radius is 2. [Answer may be left in terms of π.] 11.....

12 Express the total area of a regular octahedron in terms of its edge e. [Answer may be left in radical form.] 12.....

[OVER]

Directions (13–17): If the blank space in each of the following statements is replaced by one of the words *always, sometimes* or *never,* the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

13 If three sides of a convex spherical quadrilateral are 80°, 90° and 100°, the fourth side is ... less than 90°. 13.....

14 If two sides of a spherical triangle are equal, the angle formed by the equal sides is ... measured by the third side. 14.....

15 If a section of a circular cone made by a plane passing through an element is an isosceles triangle, the cone is ... a right circular cone. 15.....

16 The locus of points on a sphere equally distant from two points on the sphere is ... a small circle of the sphere. 16.....

17 If two angles of a spherical triangle are equal to two angles of another spherical triangle, then the third angles are ... equal. 17.....

Directions (18–20): Indicate the correct completion for *each* of the following by writing on the line at the right the letter *a, b* or *c.*

18 The locus of points a given distance from a straight line and also equidistant from two parallel planes is *never* (*a*)a point (*b*)a straight line (*c*)a circle 18.....

19 Given three trihedral angles *R, S* and *T.* Two face angles of *R* are 140° and 160°; of *S,* 130° and 150°; of *T,* 30° and 50°. The two trihedral angles in which the limits for the value of the third face angle are the same are (*a*) *R* and *S* (*b*) *S* and *T* (c) *R* and *T* 19.....

20 If angle *DEF* is the plane angle of a dihedral angle whose edge is *AB,* then *DE* is always perpendicular to (*a*)*EF* (*b*)one of the faces of the dihedral angle (*c*)*AB* 20.....