The University of the State of New York

312TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, June 21, 1951 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish this part before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry, (*c*) author of textbook used.

The minimum time requirement is four or five recitations a week for half a school year.

Part II

Answer two questions from part II.

21 Prove that in two polar triangles, each angle of one has the same measure as the supplement of the side lying opposite to it in the other. [10]

22 If line a is perpendicular to plane m, and line b, not in plane m, is perpendicular to a, prove that b is parallel to m. [10]

23 Prove that if a pyramid is cut by a plane parallel to its base, the section is a polygon similar to the base. [10]

24 Given plane M and line l parallel to M and d distance from M.

a Describe fully the locus of the centers of spheres of radius r which are

(1) tangent to plane M [2]

(2) tangent to line l [2]

b Describe fully the locus of points satisfying both conditions in *a* if

(1) $d = 5$ and $r = 3$ [2]

(2) $d = 6$ and $r = 3$ [2]

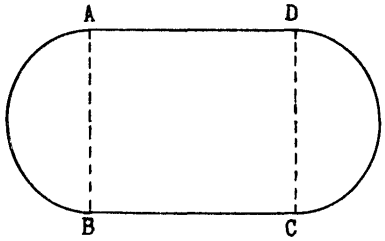
c Is the following statement true or is it false?

If d is greater than $2r$ there can be no points which satisfy both conditions stated in *a*. [2]

Part III

Answer three questions from part III.

25 The surface of a wading pool is in the shape of a rectangle with semicircular ends as illustrated in the diagram. The length AD of the rectangle is 24 feet and the diameter AB of the semicircle is 20 feet. A concrete walk 3 feet wide and $\frac{1}{2}$ foot thick is to be built around the pool. Find, to the *nearest cubic yard,* the amount of concrete required. [10]



26 A zone and an equiangular spherical triangle on the same sphere are equal in area. If the radius of the sphere is 12 inches and the altitude of the zone is 5 inches, find the number of degrees in each angle of the spherical triangle. [10]

27 A sphere is circumscribed about a rectangular parallelepiped.

a If the volume of the parallelepiped is 972 cubic inches and the edges are in the ratio 2:3:6, find the diameter of the sphere. [8]

b Find the volume of the sphere. [Use $\pi = \frac{22}{7}$] [2]

28 The slant height of a regular hexagonal pyramid makes with its base an angle θ.

a If the base edge of the pyramid is e, show that the volume of the pyramid is given by the formula $V = \frac{3}{4}e^3 \tan \theta$ [7]

b Using the formula given in *a*, find V to the *nearest tenth* if $e = 2.7$ and $\theta = 50°$ [3]

Fill in the following lines:

Name of pupil..... .....Name of school.....

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 The perimeter of a right section of a prism is 16 and the lateral edge is 20. Find the lateral area of the prism. 1.....

2 The altitude of a prism is 9 and its base is a right triangle whose legs are 3 and 4. Find the volume of the prism. 2.....

3 If the total area of a cube is 150, find its volume. 3.....

4 The slant height of a frustum of a cone of revolution is 15. If the radii of the bases are 2 and 14, find the lateral area. [Answer may be left in terms of π.] 4.....

5 A cylinder and a cone have equal volumes and equal altitudes. If the area of the base of the cylinder is 18, find the area of the base of the cone. 5.....

6 The lateral faces of a square pyramid make with the base an angle of 45°. If the altitude of the pyramid is h, express the volume of the pyramid in terms of h. 6.....

7 A plane parallel to the base of a cone cuts the altitude in a point 4 inches from the vertex. If the altitude is 10 inches and the area of the section thus formed is 20 square inches, find the area of the base. 7.....

8 The slant height s of a right circular cone is equal to the diameter of the base. Express the lateral area of the cone in terms of s. 8.....

9 The area of a lune is two-fifths the area of the sphere. Find the number of degrees in the angle of the lune. 9.....

10 The volumes of two similar cylinders of revolution are in the ratio 8:27. If the base of the larger cylinder is 18, find the base of the smaller cylinder. 10.....

11 A line 10 inches long is inclined at an angle of 43° to a plane. Find, to the *nearest tenth of an inch*, the length of its projection on the plane. 11.....

Directions (questions 12–15) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a*, *b* or *c*.

12 If a line is oblique to a plane, the number of lines that can be drawn in that plane perpendicular to the line at the point of intersection is (*a*)none (*b*)one (*c*)more than one 12.....

13 The northern hemisphere of the earth is divided into two zones approximately equal in area by the parallel of latitude at (*a*)30°N (*b*)45°N (*c*)60°N 13.....

14 The number of points which are equally distant from all points on a given circle and also a given distance from the center of that circle is (*a*)one (*b*)two (*c*)more than two 14.....

15 If one exterior angle is formed at each vertex of a convex spherical quadrilateral, the sum of these angles will be (*a*)more than 360° (*b*)less than 360° (*c*)equal to 360° 15.....

Directions (questions 16–20) — In *each* of the following, if the statement is *always* true, write *true* on the line at the right; if it is *not always* true, write *false*.

16 If two face angles of a trihedral angle are 120° and 80°, then the third angle is more than 40° and less than 160° and may have any value between these limits. 16.....

17 If angles A and B of spherical triangle ABC are right angles, then C is a vertex of the polar triangle of ABC. 17.....

18 If three lines are mutually perpendicular, any one of them is perpendicular to the plane determined by the other two. 18.....

19 If two parallel lines are parallel to plane M, they determine a plane that is parallel to plane M. 19.....

20 The opposite trihedral angles of an oblique parallelepiped are always symmetric. 20.....