The University of the State of New York

309TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, June 22, 1950 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish this part before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry, (*c*) author of textbook used.

The minimum time requirement is four or five recitations a week for half a school year.

Part II

Answer two questions from part II.

21 Prove that a line perpendicular to one of two parallel planes is perpendicular to the other also. [10]

22 Triangle ABC has a right angle at C. Line segment AD is drawn perpendicular to plane ABC. Points E and F are the midpoints of line segments DC and DB respectively. Prove that line EF is perpendicular to the plane ADC. [10]

23 Prove that a spherical angle is measured by the arc of the great circle described from its vertex as a pole and included between its sides, produced if neceessary. [10]

24 Prove that two lines which intersect two given skew lines in four distinct points can not be coplanar. [10]

Part III

Answer three questions from part III.

25 At a banquet for 70 people, tomato juice is served in glasses having the shape of a frustum of a right circular cone. The inside diameter of the bottom of the glass is $1\frac{1}{2}''$. If the depth of the juice is $3''$ and the diameter of the upper surface of the liquid is $2''$, find, to the *nearest integer,* the number of quarts used. [1 quart contains $57\frac{3}{4}$ cubic inches.] [Use $V = \frac{1}{3}\pi h\,(r_1^2 + r_2^2 + r_1 r_2)$ and $\pi = \frac{22}{7}$.] [10]

26 *a* The sides of a spherical triangle are 100°, 75° and 85°. Find the number of square inches in the area of the polar triangle if the radius of the sphere is 12 inches. [Answer may be left in terms of π.] [7]

b A zone on the same sphere has an area equal to the area of the polar triangle found in answer to part *a*. Find the height of the zone. [3]

27 Find, to the *nearest pound,* the weight of 4 feet of lead pipe which is 2 inches in inside diameter and $\frac{1}{4}$ inch thick. Lead weighs 708 pounds per cubic foot. [Use $\pi = 3.14$.] [10]

28 Each face angle at the vertex of a regular triangular pyramid is θ and each base edge is e.

a Show that the lateral area S of the pyramid is given by the formula:

$$S = \frac{3e^2}{4 \tan \frac{\theta}{2}} \qquad [6]$$

b Find S to the ***nearest square inch*** if $e = 6.7$ inches and $\theta = 56°$. [4]

Fill in the following lines:

Name of pupil .....Name of school .....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed.

1 Find the length of a diagonal of a rectangular parallelepiped whose dimensions are 3, 4 and 12. 1.....

2 Express the lateral area of the frustum of a regular square pyramid in terms of its base edges a and b and its slant height s. 2.....

3 Find the number of degrees in the sum of all the face angles of a regular octahedron. 3.....

4 Plane R intersects plane S in line m, forming an $\angle$ of 60°. Point P in R is 12 inches from m. Find the distance of P from plane S. [Answer may be left in radical form.] 4.....

5 The altitude of a cylinder of revolution is twice the radius of the base. Find the ratio of its lateral area to its total area. 5.....

6 Two similar cones of revolution have volumes in the ratio 1 : 64. Find the ratio of the radii of their bases. 6.....

7 Find the volume of a sphere whose radius is 3. [Answer may be left in terms of π.] 7.....

8 Find the radius of the sphere on which a lune with an angle of 40° has an area of 16π square inches. 8.....

9 The altitude of a pyramid is 6 inches and the base is a right isosceles triangle with legs of 6 inches. Find the volume of the pyramid. 9.....

Directions (questions 10–14) — Indicate the correct answer to each question by writing on the line at the right the letter a, b or c.

10 The locus of points at a given distance from a given line is (a)two lines (b)a cylindrical surface (c)two planes 10.....

11 If the radius of the upper base of the frustum of a right circular cone is half the radius of the lower base, the slant height is (a)shorter than (b)equal to (c)longer than the radius of the upper base. 11.....

12 The face angles of a trihedral angle may be (a) 40°, 70°, 110° (b) 100°, 120°, 150° (c) 70°, 100°, 120° 12.....

13 If three angles of a spherical quadrilateral are each equal to 80°, the fourth angle is (a)less than (b)equal to (c)greater than 120°. 13.....

14 Given two points, A and B, 6 inches apart. The locus of points 6 inches from both A and B is (a)a straight line (b)a circle (c)a plane 14.....

Directions (questions 15–20) — If the blank space in each statement is filled by one of the words, *always, sometimes* or *never,* the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

15 If plane R intersects planes S and T in two parallel lines, then planes S and T are ... parallel. 15.....

16 Through a given point it is ... possible to construct a plane perpendicular to each of two given intersecting planes. 16.....

17 The projection of a square on a plane oblique to the plane of the square is ... a rectangle. 17.....

18 A line which is perpendicular to a tangent to a circle at the point of tangency is ... perpendicular to the plane of the circle. 18.....

19 A line parallel to one of two skew lines is ... parallel to the other. 19.....

20 If spherical triangle I is congruent to spherical triangle II and is symmetric to spherical triangle III, then triangle II is ... symmetric to triangle III. 20.....