The University of the State of New York

303D High School Examination

SOLID GEOMETRY

Thursday, June 24, 1948 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish this part before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry, (*c*) author of textbook used.

The minimum time requirement is four or five recitations a week for half a school year.

Part II

Answer two questions from part II.

21 Prove that if two lines are cut by three parallel planes, the corresponding segments are proportional. [10]

22 Prove that if a point on a sphere is at a quadrant's distance from each of two other points on the sphere, not the extremities of a diameter, it is the pole of the great circle through these points. [10]

23 Given regular tetrahedron $ABCD$ with M the mid-point of BD. If lines AM and CM are drawn, prove that angle AMC is the plane angle of dihedral angle A-BD-C. [10]

24 Prove that if a plane is perpendicular to a line in one of two parallel planes, it is perpendicular to the other plane. [10]

*25 *a* Define prismatoid. [3]

b Considering the prism and the pyramid as special cases of the prismatoid, show how the formula for the volume of *each* can be obtained from the formula

$$V = \frac{h}{6}(B + B' + 4M) \quad [3, 4]$$

* This question is based on one of the optional topics in the syllabus and may be used in either part II or part III.

Part III

Answer three questions from part III.

26 A regular hexagonal prism and a right circular cylinder each have altitudes of 20 feet. An edge of the base of the prism is 2 feet and the radius of the cylinder is 1 foot. Find to the *nearest cubic foot* the difference between their volumes. [Use $\pi = 3.14$] [10]

27 A granite marker has the form of a frustum of a regular square pyramid surmounted by a regular pyramid. The base of the pyramid coincides with the smaller base of the frustum. The edges of the upper and lower bases of the frustum are 16 inches and 24 inches respectively, the altitude is 30 inches and the altitude of the pyramid is 12 inches. Find to the *nearest pound* the weight of the marker if granite weighs 180 pounds per cubic foot.

$$\left[\text{Volume of frustum} = \frac{h}{3}\ (B_1 + B_2 + \sqrt{B_1 B_2})\right] \quad [10]$$

28 A spherical ball having a diameter of 10 inches rests on a circular ring whose inner diameter is 8 inches. What fractional part of the surface of the sphere is above the ring? [10]

29 A piece of tin in the form of a sector of a circle of radius 15 is rolled into a cone so that the length of the arc of the sector becomes the circumference of the base of the cone. The angle of the sector is 216°. Find the volume of the cone. [Answer may be left in terms of π.] [10]

Fill in the following lines:

Name of pupil.....Name of school.....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 If a line is oblique to a plane, how many lines are there in the plane which are perpendicular to that line? 1.....

2 A point is equidistant from the faces of a 60° dihedral angle and is 8 inches from the edge of the angle. How far is the point from either face of the angle? 2.....

3 The bases of a frustum of a right circular cone have radii of 8 inches and 10 inches. Find the lateral area if the slant height is 4 inches. [Answer may be left in terms of π.] 3.....

4 The lateral area of a cylinder of revolution is 240π and its altitude is 10 inches. Find the radius of the base. 4.....

5 An equilateral triangle whose side is s is revolved about one of its altitudes as an axis. Express in terms of s the lateral area of the cone thus formed. 5.....

6 How many degrees are there in an angle of a lune whose area is one twelfth the area of the sphere? 6.....

7 The projections of two parallel line segments on a plane are in the ratio 7 : 5. If the longer segment is 21 inches, find the length of the shorter segment. 7.....

8 A diagonal of a cube is $5\sqrt{3}$. Find an edge. 8.....

9 If the radius of a right circular cylinder is doubled without changing its altitude, by what number is its volume multiplied? 9.....

10 The angles of a spherical triangle are 70°, 80° and 100°. How many spherical degrees are there in the area of the triangle? 10.....

11 Find the radius of the sphere whose volume is 36π. 11.....

12 The altitude of a pyramid is 20 inches. Its base is a square each of whose sides is 5 inches long. Find the area of the section made by a plane parallel to the base and 8 inches from the vertex. 12.....

Directions (questions 13–15) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a*, *b* or *c*.

13 Which of the following sets of angles may be the face angles of a trihedral angle? (*a*) 110°, 120°, 140° (*b*) 50°, 130°, 80° (*c*) 100°, 70°, 60° 13.....

14 Which of the following sets of angles may be the angles of a spherical triangle? (*a*) 175°, 185°, 190° (*b*) 50°, 60°, 70° (*c*) 60°, 90°, 100° 14.....

15 The locus of points equidistant from the sides of a triangle is (*a*) a point (*b*) a line (*c*) a plane 15.....

Directions (questions 16–20) — In each of the following, if the statement is *always* true, write the word *true* on the line at the right; if it is *not always* true, write the word *false*.

16 If a plane is passed cutting all the lateral edges of a parallelepiped, the section thus formed is a parallelogram. 16.....

17 If each of two lines is perpendicular to a third line, then the three lines are in the same plane. 17.....

18 Two lines each parallel to a given plane are parallel to each other. 18.....

19 The sides of a spherical triangle are the supplements of the opposite angles of its polar triangle. 19.....

20 If two spherical triangles on the same sphere have three sides of one equal respectively to three sides of the other, the triangles are congruent. 20.....