The University of the State of New York

300TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, June 19, 1947 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish part I before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry, (*c*) author of textbook used.

The minimum time requirement is four or five recitations a week for half a school year.

Part II

Answer two questions from part II.

21 Prove that if the first of two spherical triangles is the polar triangle of the second, the second is the polar triangle of the first. [10]

22 Plane M and plane N are both parallel to line AB. Plane P, containing AB, intersects plane M in line CD and plane N in line EF. Prove that CD is parallel to EF. [10]

23 Prove that if each of two intersecting planes is perpendicular to a third plane, their intersection is also perpendicular to that plane. [10]

*24 Using Cavalieri's theorem, show that the volume of any parallelepiped is equal to the product of its base and its altitude. [10]

* This question is based on one of the optional topics in the syllabus.

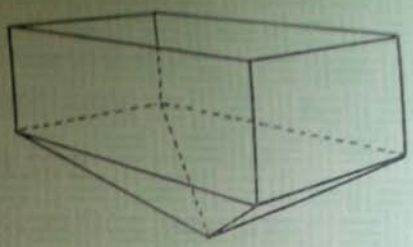
Part III

Answer three questions from part III.

25 An iron casting in the shape of the frustum of a right circular cone is 4 feet high. One base is 10 inches in diameter and the other base is 6 inches in diameter. The casting is melted and formed into a bar in the shape of a rectangular parallelepiped 4 inches wide and 2 inches thick. Find the length of the bar. [The volume of the frustum of a right circular cone is given by the formula $V = \frac{1}{3}\pi h\ (r_1^2 + r_2^2 + r_1 r_2)$] [10]

26 Two angles of a spherical triangle, on a sphere whose radius is 12 feet, are 115° and 95°. Find the third angle if the area of the triangle is 352 square feet. [Use $\pi = \frac{22}{7}$] [10]

27 A bin consists of two parts as shown in the drawing. The upper part has the form of a rectangular solid whose height is 5 feet. The lower part has the form of a pyramid whose height is 4 feet. The common base is 16 feet long and 9 feet wide. Find, correct to the *nearest bushel*, the capacity of the bin. [Allow $1\frac{1}{4}$ cubic feet for each bushel.] [10]



28 In an isosceles triangle one of the equal sides a makes with the base an angle θ.

a Show that the volume V of the solid formed by revolving the triangle through 360° about the base as an axis is given by the formula $V = \frac{2}{3}\pi a^3 \sin^2\theta \cos\theta$. [6]

b Find V correct to the *nearest integer* when $a = 6$ and $\theta = 42°$ [Use $\pi = 3.14$] [4]

Fill in the following lines:

Name of school.....Name of pupil.....

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 The volume of a sphere is $\frac{500}{3}\pi$. Find the radius of the sphere. 1.....

2 Find the total area of a right circular cone whose radius is 4 and whose slant height is 5. [Answer may be left in terms of π.] 2.....

3 Find the lateral area of a right circular cylinder whose radius is 4 and whose altitude is 9. [Answer may be left in terms of π.] 3.....

4 A zone whose altitude is 2 is on a sphere whose radius is 10. Find the area of the zone. [Answer may be left in terms of π.] 4.....

5 The volume of a circular cone is 80π and the radius is 4. Find the altitude. 5.....

6 The base edges of a frustum of a regular square pyramid are 3 and 7 and the slant height is 6. Find the lateral area of the frustum. 6.....

7 The radius of a circular cylinder is r and its altitude is $3r$. Express the volume of the cylinder in terms of r. 7.....

8 Express the total area of a regular tetrahedron in terms of its edge e. 8.....

9 A diagonal of a cube is $a\sqrt{3}$. Express the volume of the cube in terms of a. 9.....

10 Find the angle of a lune equal in area to a spherical triangle on the same sphere, if the angles of the triangle are 90°, 110° and 116°. 10.....

11 Each angle of a spherical triangle is 75°. Find in degrees the perimeter of its polar triangle. 11.....

12 A line segment 6 inches in length is inclined to a plane at an angle of 40°. Find, correct to the *nearest tenth*, the length of its projection on the plane. 12.....

13 A plane is passed parallel to the base and 3 inches from the vertex of a pyramid whose altitude is 10 inches. Find the ratio of the volume of the pyramid thus formed to that of the original pyramid. 13.....

14 The locus of points equidistant from the vertices of a triangle is (*a*) a line, (*b*) a sphere or (*c*) a point. Which is correct (*a*), (*b*) or (*c*)? 14.....

Directions (questions 15–17) — Indicate whether each statement is true or false by writing the word *true* or *false* on the line at the right.

15 Each side of a spherical quadrilateral may be a quadrant. 15.....

16 The plane of the plane angle of a dihedral angle is perpendicular to the edge of the dihedral angle. 16.....

17 Given skew lines a and b and point P not in a or b. Through P, one plane and only one plane may be passed parallel to both a and b. 17.....

Directions (questions 18–20) — If the blank in each statement is replaced by one of the words *always*, *sometimes* or *never*, the resulting statement is true. Select the word that will correctly complete *each* statement and write that word on the line at the right.

18 The lateral area of a prism is ... equal to the product of a lateral edge and the perimeter of the base. 18.....

19 Two trihedral angles are ... congruent if the three face angles of one are equal to the three face angles of the other. 19.....

20 The locus of points at a given distance from a given line and equidistant from two parallel planes is ... a circle. 20.....