The University of the State of New York

SOLID GEOMETRY

Thursday, June 21, 1945 — 9.15 a. m., to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish this part before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in solid geometry, (c) author of textbook used.

The minimum time requirement is five recitations a week for half a school year.

Part II

Answer two questions from part II.

21 Prove that if a line is perpendicular to a plane, every plane passed through the line is perpendicular to the given plane. [10]

22 Given parallel lines a and b neither of which lies in plane P; prove that if one of these lines is parallel to P, the other is also. [10]

23 Prove that a spherical angle is measured by the arc of a great circle described from its vertex as a pole and included between its sides produced if necessary. [10]

*24 Given plane R perpendicular to line l at point M; P is a moving point whose distance from R is always twice its distance from l.

    a Represent by means of a drawing the locus of point P. [8]

    b What is the name of this locus? [2]

Part III

Answer three questions from part III.

25 The sides of a spherical triangle are 106° 48', 123° 12' and 90°.

    a Find the angles of the polar triangle. [3]

    b Find the number of spherical degrees in the polar triangle. [2]

    c If the area of the polar triangle is 2π, find the radius of the sphere on which it is drawn. [5]

*This question is based on one of the optional topics in the syllabus and may be used in either part II or part III.

26 *a* The base edge of a regular square pyramid is *e* and its slant height makes with its projection on the base an angle θ. Show that the lateral area *S* of the pyramid is given by the formula

$$S = \frac{e^2}{\cos \theta} \quad [5]$$

b Using the formula given in *a*, find, correct to the *nearest dollar*, the cost of painting a church spire in which $e = 16$ feet and $\theta = 75°$. Allow 20 cents per square foot for the cost of material and labor. [5]

27 The accompanying figure represents a solid in the form of a frustum of a right circular cone from which a right circular cylinder has been removed. The radius of the cylinder is equal to the radius of the upper base of the frustum. The slant height of the frustum is 13, the radii of its bases are 9 and 4, and the height of the cylinder is 6. Find, correct to the *nearest integer,* the volume of the solid. [Use $\pi = \frac{22}{7}$] [The formula for the volume of a frustum of a circular cone is $V = \frac{1}{3} \pi h (r_1^2 + r_2^2 + r_1 r_2)$] [10]
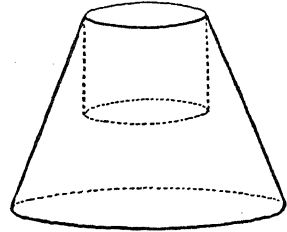
28 *a* The altitude of a prism is *h* and its base is a regular polygon of 10 sides. If a side of the polygon is *a*, show that the volume *V* of the prism is given by the formula $V = 2.5\ a^2 h \tan 72°$. [5]

b Using the formula given in *a*, find, correct to the *nearest pound*, the weight of a concrete column in which $a = 4$ inches and $h = 6$ feet. [Allow 150 pounds per cubic foot.] [5]

SOLID GEOMETRY

Fill in the following lines:

Name of school..Name of pupil..

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 Find the total area of a right circular cylinder whose radius is 3 and whose altitude is 7. [Answer may be left in terms of π.]

1................

2 Find the lateral area of a frustum of a regular triangular pyramid whose base edges are 6 and 8 and whose slant height is 5.

2................

3 Find the lateral area of a cone of revolution whose radius is 3.5 inches and whose slant height is 8.2 inches. [Use $\pi = \frac{22}{7}$]

3................

4 Find the area of a lune whose angle is 5° on a sphere whose radius is 12. [Answer may be left in terms of π.]

4................

5 How many spherical degrees are there in a spherical quadrilateral whose angles are 80°, 90°, 110° and 120°?

5................

6 The base of a pyramid is a right triangle whose legs are 6 and 10. The altitude of the pyramid is 18. Find its volume.

6................

7 The altitude of a right circular cone is 12 and its volume is 64 π. Find its radius.

7................

8 The area of a sphere is 4 π. Find its volume. [Answer may be left in terms of π.]

8................

9 An observation tower has the form of a regular square pyramid. If the tower is 100 feet high and has a base area of 1600 square feet, find the area of a platform 80 feet from the base of the tower.

9................

10 Two similar rectangles are revolved through 360° about corresponding sides as axes. If the areas of the two rectangles are in the ratio 4 to 1, find the ratio of the volumes of the two solids thus formed.

10................

11 Tell whether the converse of the following proposition is *true* or *false*: If a spherical triangle is birectangular, the triangle is isosceles.

11................

Directions (questions 12–15) — Indicate whether *each* statement is true or false by writing the word *true* or *false* on the line at the right.

12 If two zones have equal altitudes, their areas are to each other as the squares of the radii of the spheres on which they are drawn.

12................

13 If two sides of a spherical triangle are 70° and 90°, the third side may have any value between 20° and 160°.

13................

14 If two sides of a spherical triangle are unequal, the angles opposite these sides are unequal and the greater angle lies opposite the greater side.

14................

15 If the angles of a spherical triangle are given, the triangle is determined.

15................

Directions (questions 16–20) — Indicate the correct answer to *each* question by writing on the line at the right the letter, *a, b* or *c.*

16 If a plane is determined by two lines, the lines   (*a*)must be parallel (*b*)must intersect   (*c*)may intersect

16......

17 If two equal and parallel line segments are oblique to a plane, their projections on the plane are always   (*a*)equal   (*b*)parallel   (*c*)coincident

17......

18 If two planes P and Q make equal dihedral angles with a third plane, P and Q (*a*) may intersect   (*b*) must intersect   (*c*) must be parallel   18......

19 A plane is determined if   (*a*) it is perpendicular to both faces of a dihedral angle   (*b*) it passes through a given point and is parallel to the edge of a dihedral angle   (*c*) it passes through a given point and is perpendicular to the edge of a dihedral angle   19......

20 The locus of points equidistant from two given intersecting planes and at a given distance from one of these planes consists of   (*a*) two lines   (*b*) four lines (*c*) eight lines   20......