SOLID GEOMETRY

Thursday, June 17, 1943 — 9.15 a. m. to 12.15 p. m., only

Instructions

Part I is to be done first and the maximum time allowed for it is one and one half hours. At the end of that time, this part of the examination must be detached and will be collected by the teacher. If you finish this part before the signal to stop is given, you may begin part II.

Write at top of first page of answer paper to parts II and III (a) name of school where you have studied, (b) number of weeks and recitations a week in solid geometry, (c) author of textbook used.

The minimum time requirement is five recitations a week for half a school year.

Part II

Answer two questions from part II.

21 a Prove that if a pyramid is cut by a plane parallel to its base, the edges and altitude are divided proportionally. [8]

   b Complete the following statement: If a pyramid is cut by a plane parallel to its base, the area of the section thus formed is to the area of the base .... [2]

22 Given trihedral angle V-RST with face angles RVT and RVS equal. On VT and VS equal line segments VA and VB are laid off and AB is drawn. Prove that AB is perpendicular to the plane determined by RV and M, the mid-point of AB. [10]

23 Prove that in two polar triangles, each angle of one has the same measure as the supplement of the side lying opposite it in the other. [10]

*24 a Define prismatoid. [3]

   b Tell whether *each* of the following statements is *true* or *false*:
     (1) The bases of a prismatoid are similar. [1]
     (2) The bases of a prismatoid can not be congruent. [1]
     (3) Prismatoids with bases and mid-sections respectively equal, are to each other as their altitudes. [The formula for the volume of a prismatoid is

$$V = \frac{h}{6} (B + B' + 4m)]$$ [1]

     (4) Show how the formula for the volume of a prism can be obtained from the formula for the volume of a prismatoid. [4]

* This question is based on one of the optional topics in the syllabus.

Fill in the following lines:

Name of school..................................Name of pupil..................................

Part I

Answer all questions in part I. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–4) — Indicate the correct answer to *each* question by writing on the line at the right the letter *a*, *b* or *c*.

1 Through a line and its projection on a given plane, a plane can be passed which is (*a*) parallel to the given plane, (*b*) perpendicular to the given plane, (*c*) oblique to the given plane.

1.....................

2 Through a given point which does not lie in either of two given intersecting planes, *R* and *S*, (*a*) one and only one line can be drawn parallel to both *R* and *S*, (*b*) more than one line can be drawn parallel to both *R* and *S*, (*c*) no line can be drawn parallel to both *R* and *S*.

2.....................

3 The three face angles of a trihedral angle may be (*a*) 110°, 120° and 130°, (*b*) 15°, 12° and 18°, (*c*) 80°, 35° and 45°.

3.....................

4 The locus of points equidistant from three points not all in the same straight line is (*a*) a circle, (*b*) a line, (*c*) a point.

4.....................

Directions (questions 5–10) — If the blank in each statement is replaced by one of the words *always, sometimes* or *never*, the resulting statement will be true. Select the word that will correctly complete *each* statement and write this word on the line at the right.

5 The diagonals of a parallelepiped ... bisect each other.

5.....................

6 The locus of points at a given distance from a given line is ... a conical surface.

6.....................

7 If line *a* is skew to line *b* and *b* is skew to line *c*, then *a* is ... skew to *c*.

7.....................

8 If, on the same or equal spheres, two spherical triangles are mutually equilateral, the triangles are ... equal in area.

8.....................

9 If two angles of a spherical triangle are complementary, the triangle is ... a right spherical triangle.

9.....................

10 Sections of unequal spheres made by planes unequally distant from the centers are ... equal.

10.....................

Directions (questions 11–20) — Write the answer to *each* question on the line at the right.

11 The altitude of a regular square pyramid is equal to an edge *e* of the base. Express the volume of the pyramid in terms of *e*.

11.....................

12 The perimeter of the mid-section of a frustum of a right circular cone is *p* and the slant height of the frustum in *s*. Express the lateral area of the frustum in terms of *p* and *s*.

12.....................

13 A rectangle whose sides are 3 and 4 is revolved through 360° about its longer side as an axis. Find the total area of the resulting solid. [Answer may be left in terms of π.]

13.....................

14 The lateral areas of two similar cones of revolution are in the ratio 25 : 49 and the altitude of the larger one is 14. Find the altitude of the smaller.

14.....................

15 The total area of a cube is 6. Find a diagonal of the cube. [Answer may be left in radical form.]

15.....................

16 How many diagonals has a regular octahedron?

16.....................

[3]

17 Express the area of a sphere in terms of its radius r.

17................

18 Find the radius of a sphere whose volume is 2304π.

18................

19 Find the number of spherical degrees in a spherical triangle whose angles are 80°, 65° and 75°.

19................

20 Is the following statement true or false?

If a zone and a lune on the same or equal spheres are equal then the altitude of the zone is to the radius of the sphere as the number of degrees in the angle of the lune is to 180.

20................

[4]

170

Part III

25 A contract calls for 1200 concrete posts. The posts are to have the form of a prism whose base is a square 8 inches on a side and whose height is 6 feet. The concrete is to be made of one part cement, two parts sand and four parts gravel. Find, correct to the *nearest cubic foot*, the amount of cement needed to fulfill this contract. [10]

26 The radius of the base of a right circular cone is r and an element of the cone makes with the base an angle θ.

    a Express the lateral area S of the cone as a function of r and θ. [6]

    b Find S, correct to the *nearest integer*, if $r = 3$ and $\theta = 27°$ [Use $\pi = 3.14$] [4]

27 *a* Show that N, the number of cubic feet of metal necessary to make t feet of pipe line whose outer diameter is d feet and whose inner diameter is d' feet, is given by the formula

$$N = \frac{\pi t (d + d') (d - d')}{4} \quad [6]$$

    b Find N if $t = 1400$ feet, $d = \frac{3}{4}$ foot, and $d' = \frac{1}{2}$ foot. [Use $\pi = \frac{22}{7}$] [4]

28 War maneuvers in the vicinity of the Solomon Islands occupy a portion of the earth's surface bounded by the equator, the parallel of latitude 15° S. and the meridians of longitude 150° E. and 165° E.

    a If r represents the radius of the earth, show that the area K of this portion is given by the formula $K = \dfrac{\pi r^2 \sin 15°}{12}$ [Suggestion: The altitude of the zone whose bases are the equator and the parallel of latitude 15° S. is $r \sin 15°$.] [7]

    b Which of the following is correct: The area of this portion of the earth's surface is (1) less than 250,000 square miles, (2) approximately 500,000 square miles or (3) more than 1,000,000 square miles? [Use 4,000 miles as the radius of the earth.] [3]