The University of the State of New York

SOLID GEOMETRY

Thursday, June 18, 1942 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Part I

This part is to be done first and the maximum time allowed for it is one and one half hours.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Parts II and III

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for half a school year.

Fill in the following lines:

Name of school.. Name of pupil..

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

1 How many vertices has the solid which is bounded by the smallest number of planes?

1........................

2 One quart of paint is required to cover the surface of a given sphere. How many quarts of paint would be required to cover the surface of a sphere whose radius is double that of the given sphere?

2........................

3 Find an edge of a cube whose volume is numerically equal to its total surface.

3........................

4 Find the length of the longest rod (neglecting its thickness) that will just fit into a rectangular box whose dimensions are 2, 3 and 6.

4........................

5 Two face angles of a trihedral angle are 75° and 115°. Can the third face angle have a value of 170°? [Answer *yes* or *no*.]

5........................

6 The angles of a spheric triangle are 40°, 105° and 92°. Find the number of degrees in the longest side of its polar triangle.

6........................

7 If the earth is assumed to be a sphere, what part of its surface is represented by a lune whose angle is 18°?

7........................

8 If two zones on different spheres are equal in area, must the radii of these spheres be equal? [Answer *yes* or *no*.]

8........................

9 The area of a spheric triangle is 50 spheric degrees. Express this area in square inches, if the radius of the sphere on which it is drawn is 6 inches. [Answer may be left in terms of π.]

9........................

10 Express the total area of a cylinder of revolution in terms of its altitude h and the radius r of its base.

10........................

11 The slant height of the frustum of a regular pyramid is 10 inches and the perimeters of its bases are 12 inches and 18 inches. Find the lateral area.

11........................

12 The locus of the centers of all spheres tangent to the two faces of a dihedral angle is (a) a plane, (b) a pair of planes or (c) a line. Which is correct, (a), (b) or (c)?

12........................

13 The projection of a line segment on a plane is one half the length of the line segment. Find the number of degrees in the angle of inclination of this line with the plane.

13........................

14 A pyramid is to be cut by a plane parallel to its base so that the area of the resulting section will be one ninth the area of the base. What is the ratio of the two segments into which each lateral edge will be divided?

14........................

Directions (questions 15–20) — Indicate whether each statement is true or false by writing the word *true* or the word *false* on the line at the right.

15 All sections of a circular cone made by planes are circles.

15........................

16 If AB and CD are two straight lines in different planes, then it is impossible for AB and CD to intersect.

16........................

17 If line AB is perpendicular to lines BC and BD, then AB is perpendicular to any line drawn through B.

17........................

[3]

[OVER]

18 The volume of a sphere is equal to one third the product of its surface and its radius.

18...................

19 If a line is in neither face of a dihedral angle but is parallel to its edge, then the line is parallel to both faces of the dihedral angle.

19...................

20 The faces of a regular polyhedron may be regular octagons.
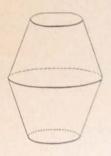
20...................

Part II

Answer two questions from this part.

21 Prove that all the perpendiculars that can be drawn to a given line at a given point lie in the plane perpendicular to the line at the point. [10]

22 The plane of a parallelogram is oblique to a given plane. Prove that the projection of the parallelogram upon the given plane is also a parallelogram. [10]

23 Prove that every section of a sphere made by a plane is a circle. [10]

*24 Cylinders and spheres may be considered as special cases of the prismatoid. The formula for the volume of a prismatoid is $V = \dfrac{h}{6}(B + B' + 4m)$. Using this formula, find the volume of

    a A cylinder whose altitude is h and the radius of whose base is r [5]

    b A sphere whose radius is r [5]

Part III

Answer three questions from this part.

25 In rectangle $ABCD$, side $AD = 12$ inches and diagonal $AC = 13$ inches. The figure is revolved through $360°$, using side AD as an axis.

    a Find the volume of the solid generated by rectangle $ABCD$. [4]

    b Find the volume of the solid generated by triangle ADC. [3]

    c Find the volume of the solid generated by triangle ABC. [3]

26 Each lateral edge of a regular triangular pyramid is 5.0 feet and the altitude of the pyramid is 3.0 feet.

    a Find, correct to the *nearest tenth of a foot*, a base edge of the pyramid. [6]

    b Find, correct to the *nearest degree*, the dihedral angle formed by the base and a lateral face. [4]

27 A solid cylindric lead bar 36 inches long and 1 inch in diameter is melted and molded into solid spheric shot each .32 inch in diameter. How many shot can be made? [10]

28 The solid represented at the right consists of two congruent frustums of right circular cones.

    a Sketch the drawing on your paper. State what measurements should be taken to determine the lateral area and indicate them on your drawing. [4]

    b Represent these measurements by letters and write a formula for the lateral area in terms of these letters. [6]

    * This question is based on one of the optional topics in the syllabus.

[2]