The University of the State of New York

281st High School Examination

SOLID GEOMETRY

Thursday, June 19, 1941 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Part I

This part is to be done first and the maximum time allowed for it is one and one half hours.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Parts II and III

Write at top of first page of answer paper to parts II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for half a school year.

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Part I

Answer all questions in this part. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–9) — Write on the dotted line at the right of each question the expression which, when inserted in the corresponding blank, will make the statement true.

1 A side of a spheric triangle contains 50 degrees. The angle opposite this side in the polar triangle contains ... degrees. 1.....

2 The angle of a lune is 45°. The area of the sphere on which it is drawn is ... times the area of the lune. 2.....

3 The edges of two cubes are in the ratio 3:1. The volume of the larger cube is ... times the volume of the smaller. 3.....

4 The total area T of a cone of revolution whose slant height is l and whose radius is r is given by the formula $T =$ .... 4.....

5 If each edge of a tetrahedron is 8 inches, its total area is ... square inches. [Answer may be left in radical form.] 5.....

6 If the area of a right section of a prism is 14 square inches and the lateral edge is 7 inches, the volume of the prism is ... cubic inches. 6.....

7 The area of a sphere is 36π square inches. The volume of this sphere is ... cubic inches. [Answer may be left in terms of π.] 7.....

8 An element of a right circular cone is 10 inches and makes an angle of 60° with the plane of the base. The radius of the base is ... inches. 8.....

9 A zone is drawn on a sphere whose radius is 8. If the area of the zone is 32π, the altitude of the zone is .... 9.....

Directions (questions 10–15) — Indicate whether each statement is *always* true, *sometimes* true, or *never* true by writing on the dotted line at the right the word *always*, *sometimes* or *never*.

10 Through a given point on a sphere a great circle can be drawn whose plane is parallel to the plane of a given great circle of this sphere. 10.....

11 A diagonal of a face of a cube is perpendicular to a diagonal of the cube. 11.....

12 An edge of a cube is perpendicular to each of four other edges and also to two face diagonals. 12.....

13 If the section of a solid made by a plane is a circle, then the solid is a sphere. 13.....

14 Two lines determine a plane. 14.....

15 If a line is drawn in one face of a dihedral angle perpendicular to the edge, it is perpendicular to the other face. 15.....

16 The sum of the angles of a spheric quadrilateral is 400°. Find the spheric excess. 16.....

17 A cylinder and a cone have equal bases and equal altitudes. Find the ratio of their volumes. 17.....

Directions (questions 18–20) — Write on the dotted line at the right the expression *increases*, *decreases* or *remains unchanged*, which, when inserted in the corresponding blank, will make the statement true.

These questions refer to a regular pyramid whose base remains unchanged while its vertex moves nearer and nearer to the base.

18 The volume of the pyramid .... 18.....

19 The sum of the face angles at the vertex .... 19.....

20 The area of the mid-section .... 20.....

See instructions for parts II and III on page 1.

Part II

Answer two questions from this part.

21 Prove that if two lines are cut by three parallel planes, the corresponding segments are proportional. [10]

22 The following theorems refer to triangles on a plane. For *each* theorem write the corresponding one referring to triangles on a sphere. [10]

a The sum of the angles of a triangle is 180°.

b The sum of two sides of a triangle is greater than the third side.

c The area of a triangle is equal to one half the product of its base and its altitude.

d If the three sides of one triangle are equal respectively to the three sides of another, the triangles are congruent.

e If the three angles of one triangle are equal respectively to the three angles of another, the triangles are similar.

23 Prove that the plane determined by one edge of any triangular pyramid and the mid-point of the opposite edge bisects the pyramid. [10]

*24 Represent by a drawing and describe *each* of the following loci:

a The locus of points equidistant from all points of a circle [4]

b The locus of points at a given distance d from a circle whose radius is r, where d is less than r [6]

Part III

Answer three questions from this part.

25 *a* A housekeeper has the choice of buying oranges which measure 2 inches in diameter for 30 cents a dozen or oranges which measure 3 inches in diameter for 40 cents a dozen. If 30 cents a dozen is a fair price for the smaller oranges and the two kinds of oranges are of the same quality, show that the price at which the larger oranges should be sold is approximately $1 a dozen. [5]

b Tomato juice is sold in cans of two different sizes. One can is 6 inches in height and 3 inches in diameter and the other is 7 inches in height and 6 inches in diameter. Which is the more economical to buy for the same amount of money, 4 of the smaller size or one of the larger size? [Show work necessary to obtain your answer.] [5]

26 Points A, B and C are the vertices of a right triangle whose hypotenuse is AB.

a What is the locus of points at a given distance d from A? [2]

b What is the locus of points equidistant from points B and C? [2]

c If d is greater than one half of BC, what is the locus of points satisfying the conditions given in both *a* and *b*? [4]

d Under what condition would there be only one point satisfying the conditions in both *a* and *b*? [2]

27 How many cubic yards of earth must be used to make a railroad embankment 300 feet long, if a cross section of the embankment is an isosceles trapezoid whose bases are 20 feet and 36 feet and whose legs are 17 feet each? [10]

28 The faces of a regular square pyramid are equilateral triangles 12 inches on a side.

a Find the slant height of the pyramid. [2]

b Which makes the larger angle with the base, the lateral edge or the slant height? Find, correct to the *nearest degree*, the difference between these two angles. [8]

* This question is based on one of the optional topics in the syllabus.

170