SOLID GEOMETRY

Thursday, June 22, 1939 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for half a school year.

Fill in the following lines:

Name of school..Name of pupil...

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–7) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1 The lateral area of a right circular cylinder circumscribed about a sphere whose radius is 2 is .... [Answer may be left in terms of π.]

1...............

2 The volume V of a sphere whose radius is r is given by the formula $V = ...$

2...............

3 The angles of a spheric triangle are 100°, 70° and 90°. The number of spheric degrees in the area of this triangle is ....

3...............

4 The areas of two lunes drawn on the same sphere are in the ratio 1:4. If the angle of the smaller lune is 10°, the angle of the larger lune is ....

4...............

5 A prism and a pyramid have equal volumes and equal bases. If the altitude of the prism is h, the altitude of the pyramid is ....

5...............

6 The altitude of a frustum of a pyramid is 20″. If a plane parallel to a base of the frustum divides a lateral edge of the frustum into segments which are in the ratio 2:3, it divides the altitude into segments the longer of which is ... inches.

6...............

7 The lateral area of a prism is equal to the product of a lateral edge and the perimeter of a ....

7...............

Directions (questions 8–15) — Indicate the correct answer to each question by writing the letter *a*, *b* or *c* on the dotted line at the right.

8 The sum of the sides of a convex spheric polygon is (*a*) less than 360°, (*b*) equal to 360° or (*c*) greater than 360°.

8...............

9 The face angles of a trihedral angle may be (*a*) 50°, 40°, 90°; (*b*) 170°, 110°, 90° or (*c*) 60°, 50°, 40°.

9...............

10 A sphere can be circumscribed about (*a*) any parallelepiped, (*b*) any right parallelepiped or (*c*) any rectangular parallelepiped.

10...............

11 If a polyhedron has 4 vertices and 6 edges, it has (*a*) 4 faces, (*b*) 6 faces or (*c*) 8 faces.

11...............

12 If the lateral areas of two similar solids are in the ratio 4:1, the volumes are in the ratio (*a*) 16:1, (*b*) 8:1 or (*c*) 64:1.

12...............

13 A plane is determined if (*a*) it passes through a given point and is perpendicular to a given plane, (*b*) it passes through a given point and is parallel to a given line or (*c*) it passes through a given point and is perpendicular to a given line.

13...............

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14 If the length of a parallel of latitude on the surface of the earth is one half the length of the equator, its angular distance north or south of the equator is approximately (a) 30°, (b) 45° or (c) 60°.

14.................

15 The projection of a circle on a plane (a) may be a straight line segment, (b) is always a circle or (c) is never a circle.

15.................

Directions (questions 16–20) — Indicate whether each statement is *always* true, *sometimes* true or *never* true by writing on the dotted line at the right the word *always, sometimes* or *never.*

16 If two lines are not in the same plane, a plane can be passed through one of these lines parallel to the other.

16.................

17 Planes P and Q are parallel and plane R intersects plane P. The locus of points which are equally distant from P and Q and also at a given distance d from R consists of four parallel lines.

17.................

18 If two planes P and Q form equal dihedral angles with a third plane, then P and Q are parallel.

18.................

19 The plane of the plane angle of a dihedral angle is perpendicular to the faces of the dihedral angle.

19.................

20 Two trihedral angles are either congruent or symmetric if the three dihedral angles of one are equal to the three dihedral angles of the other.

20.................

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See instructions for groups II and III on page 1.

Group II

Answer three questions from this group.

21 Prove that through a given point in a given plane one line and only one can be drawn perpendicular to the given plane. [10]

22 Prove that in two polar triangles each angle of one has the same measure as the supplement of the side lying opposite to it in the other. [10]

23 Given two lines m and n in the same plane and two points A and B on n
   a What is the locus of points which are equidistant from A and B? [2]
   b What is the locus of points which are at a given distance d from m? [2]
   c If m and n are parallel, what is the locus of points which satisfy both conditions given in a and b? [3]
   d If the locus of points which satisfy both conditions given in a and b consists of two parallel lines, what then is the relation between m and n? [3]

24 Prove that if a line intersects two parallel planes, it is equally inclined to the planes. [10]

*25 Using Cavalieri's Principle, prove that two triangular pyramids having equal bases and equal altitudes are equal. [10]

Group III

Answer two questions from this group.

26 An isosceles trapezoid whose bases are 10 and 24 and one of whose base angles is 60° is revolved through 180° about the line which passes through the mid-points of the bases. Find the total area of the resulting solid. [Answer may be left in terms of π.] [10]

27 Through a metal sphere a cylindrical hole 12.0 inches in diameter is bored. The axis of the cylinder coincides with a diameter of the sphere. The depth of the hole is 16.0 inches. Find, correct to the *nearest tenth of a square inch*, the area of the part of the sphere which remains. [Use $\pi = 3.14$] [10]

28 The slant height s of a right circular cone makes with its projection on the base an angle A.
   a Express the volume V of the cone as a function of s and A. [5]
   b Find, correct to the *nearest cubic inch*, the value of V, if s equals 2.50 inches and A equals 40°. [Use $\pi = 3.14$] [5]

* This question is based on one of the optional topics in the syllabus.

[2]