The University of the State of New York

272D HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Thursday, June 23, 1938 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for half a school year.

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Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–14) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1 If a pyramid is cut by a plane parallel to the base, the section is a polygon ... to the base. 1.....

2 If the radius of a sphere is multiplied by 2, the surface of the sphere is multiplied by .... 2.....

3 Through a given point outside a given plane one plane and only one can be passed ... to the given plane. 3.....

4 If a straight line segment is perpendicular to a given plane, its projection on the plane is a .... 4.....

5 The lateral edge of a regular pyramid makes with its projection on the base an angle of 52°. If the length of the projection is 10 inches, the height of the pyramid, correct to the *nearest inch*, is ... inches. 5.....

6 The total area T of a right circular cylinder, expressed as a function of its radius r and its height h, is $T =$ .... 6.....

7 The volumes of two similar cones are 8 and 125 respectively. If the height of the larger cone is 10, the height of the smaller cone is .... 7.....

8 The radius of a sphere whose volume is 36π cubic inches is ... inches. 8.....

9 On a sphere of radius 2 feet a zone is drawn whose height is 1 foot. The area of the zone is ... square feet. [Answer may be left in terms of π.] 9.....

10 If the area of a lune is $\frac{1}{12}$ the area of the sphere on which it is drawn, the number of degrees in the angle of the lune is .... 10.....

11 A plane parallel to the edge of a dihedral angle intersects the two faces. The lines of intersection are ... to each other. 11.....

12 If the edges of a rectangular parallelepiped are a, b and c, then the diagonal d, expressed as a function of a, b and c, is $d =$ .... 12.....

13 If the sum of the perimeters of the bases of a frustum of a regular pyramid is k and the slant height of the frustum is l, the lateral area S of the frustum, expressed as a function of k and l, is $S =$ .... 13.....

14 The sum of the angles of a spheric quadrilateral is less than eight right angles and greater than ... right angles and may have any value between these limits. 14.....

Directions (questions 15–20) — Indicate whether each of the following statements is *always* true, *sometimes* true, or *never* true, by writing the word *always*, *sometimes* or *never* in the space at the right.

15 The sum of the sides of a convex spheric quadrilateral is less than 360°. 15.....

16 Two spheric triangles which are symmetric to a third triangle are symmetric to each other. 16.....

17 If a point is equidistant from two points on a small circle of a sphere, it is a pole of that circle. 17.....

18 A sphere can be circumscribed about a parallelepiped. 18.....

19 The locus of points which are 5 inches from each of two given points 6 inches apart is a circle of radius 4 inches whose plane is perpendicular to the line segment joining the two points at its midpoint and whose center is the midpoint of this line segment. 19.....

20 If the edge of a cube is increased by a, the volume of the cube is increased by a^3. 20.....

See instructions for groups II and III on page 1.

Group II

Answer three questions from this group.

21 Prove that if each of two intersecting planes is perpendicular to a third plane, their intersection is also perpendicular to that plane. [10]

22 Prove that the sum of any two face angles of a trihedral angle is greater than the third face angle. [10]

23 Prove that if a line not lying in either of two parallel planes is parallel to one of these planes, it is parallel to the other also. [10]

24 Prove that the lateral area of a regular pyramid is equal to one half the product of its slant height and the perimeter of its base. [10]

25 A prism and a pyramid have equal altitudes, equal volumes and similar bases. Prove that the ratio of a base edge of the pyramid to a corresponding base edge of the prism is $\sqrt{3} : 1$. [10]

*26 Given a plane R and a point P four inches from R

a What is the locus of points which are a given distance d from R? [3]

b What is the name of the locus of points which are equally distant from P and R? [3]

c What is the locus of points which will satisfy both conditions given in *a* and *b* if d is 4 inches? [3]

d How many points are there which will satisfy both conditions given in *a* and *b* if d is 2 inches? [1]

Group III

Answer two questions from this group.

27 The sides of a spheric triangle drawn on a sphere whose radius is 12.0 inches are 70°, 100° and 90°. Find, correct to the *nearest tenth of a square inch*, the area of the polar triangle. [Use $\pi = 3.14$] [10]

28 The hypotenuse of a right triangle is s and one of the acute angles is x. A solid is generated by revolving the triangle through 360° about the side opposite x as an axis.

a Express the height of the solid and the radius of its base as a function of s and x. [2]

b Express the volume V of the solid as a function of s and x. [3]

c Find V if $s = 1$ and $x = 45°$ [Answer may be left in terms of π and radicals.] [2]

d If s remains the same but x increases, what change takes place in the altitude? the radius? [2]

e As x changes from 45° to 90°, what change takes place in V? [1]

29 In building a fence it is necessary to set 48 posts in concrete. The posts have square ends, 4 inches on a side, and are to be set in concrete to a depth of 18 inches. The postholes are cylindrical in shape, 14 inches in diameter and 18 inches deep. Find the number of cubic inches of concrete needed to set the posts. [Use $\pi = \frac{22}{7}$] [10]

* This question is based on one of the optional topics in the syllabus.