Instructions

Do not open this sheet until the signal is given.

Group I

This group is to be done first and the maximum time allowed for it is one and one half hours.

If you finish group I before the signal to stop is given you may begin group II. However, it is advisable to look your work over carefully before proceeding, since *no credit will be given any answer in group I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on group I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Groups II and III

Write at top of first page of answer paper to groups II and III (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry, (*c*) author of textbook used.

The minimum time requirement is five recitations a week for half a school year.

Fill in the following lines:

Name of school...Name of pupil..

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–15) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1 All planes passed through the axis of a small circle of a sphere are ... to the plane of the small circle.   Ans.............

2 If a ... is perpendicular to one of two intersecting lines, it is never perpendicular to the other.   Ans.............

3 If a line is drawn in one face of a right dihedral angle and is perpendicular to the edge, it is ... to the other face.   Ans.............

4 The lateral area of a frustum of a cone of revolution is equal to one half the product of the ... and the sum of the circumferences of the bases.   Ans.............

5 The bases of two similar pyramids are in the ratio $9:4$; if the altitude of the smaller pyramid is 3, the altitude of the larger pyramid is ....   Ans.............

6 The area of a zone of height 3 feet drawn on a sphere of radius 7 feet is ... square feet. [Use $\pi = \frac{22}{7}$]   Ans.............

7 The edges of a rectangular parallelepiped are 3, 4 and 5 inches. The length of the diagonal of the parallelepiped, expressed in radical form, is ... inches.   Ans.............

8 A right circular cylinder is circumscribed about a sphere. If the lateral area of the cylinder is 400π, the volume of the sphere is .... [Answer may be left in terms of π.]   Ans.............

9 The base edge of a square pyramid is e and the height is h. The volume of the pyramid, expressed as a function of e and h, is $V = $ ....   Ans.............

10 The cost in dollars of gilding a hemispheric dome 21 feet in diameter at $1 per square foot is .... [Use $\pi = \frac{22}{7}$]   Ans.............

11 A spheric triangle and a lune drawn on the same sphere are equal in area. If the angles of the spheric triangle are $50°$, $80°$ and $100°$, the number of degrees in the angle of the lune is ....   Ans.............

12 The height of a certain right circular cone is equal to the radius of the base. If the volume of the cone is 72π cubic inches, the radius of the base is ... inches.   Ans.............

13 The capacity in cubic feet of a tank in the form of a right circular cylinder whose height is 21 feet and the radius of whose base is 12 feet is .... [Use $\pi = \frac{22}{7}$]   Ans.............

14 The locus of points equidistant from two intersecting lines consists of two ....   Ans.............

15 The locus of points equidistant from two intersecting planes and at a given distance from their line of intersection consists of four ....   Ans.............

[OVER]

Directions (questions 16–20) — Indicate whether each of the following statements is *always* true, *sometimes* true or *never* true by writing the word *always, sometimes* or *never* on the dotted line at the right.

16 The three face angles of a trihedral angle may be 105°, 120° and 135°.

Ans............

17 If three angles of a spheric quadrilateral are right angles, the fourth angle is obtuse.

Ans............

18 Through a given point one and only one plane can be passed parallel to each of two given lines.

Ans............

19 An angle of a spheric triangle is the supplement of the side opposite in the same triangle.

Ans............

20 If two spheres are tangent to the same line at the same point, the spheres are tangent to each other.

Ans............

See instructions for groups II and III on page 1.

Group II

Answer three questions from this group.

21 Prove that if a line is perpendicular to each of two intersecting lines at their point of intersection, it is perpendicular to the plane of the two lines. [10]

22 Prove that the sum of the angles of a spheric triangle is greater than 180° and less than 540°. [10]

23 Prove that the volume of a right triangular prism is equal to the area of any lateral face multiplied by one half the distance between that face and the opposite edge of the prism. [10]

24 Prove that if a line segment is equal to its projection on a plane, the line segment and the plane are parallel. [10]

Group III

Answer two questions from this group.

Leave all work on the paper; merely writing the answers is not sufficient. Use $\pi = \frac{22}{7}$ unless otherwise stated.

25 The sides of a spheric triangle drawn on a sphere whose radius is 5.93 inches are 80°, 100° and 120°. Find, correct to the *nearest square inch*, the area of the polar triangle. [Use $\pi = 3.14$] [10]

26 A bin for storing grain has the form of a right prism with a right triangle for its base. One leg of the right triangle is 18 feet and the adjacent acute angle is 47°. Find, correct to the *nearest bushel*, the capacity of this bin if its depth is $4\frac{1}{2}$ feet. [One bushel contains approximately $1\frac{1}{4}$ cubic feet.] [10]

27 The accompanying figure represents a cube with K the mid-point of the edge HG. The edge of the cube is 4 inches. Find the lateral area of the pyramid $K\text{-}ABCD$. [Answer may be left in radical form.] [10]
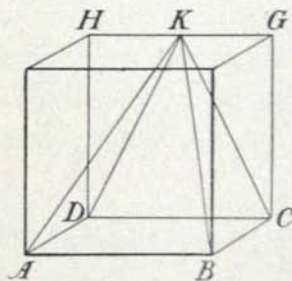
*28 Starting with the formula for the volume of a prismatoid $V = \frac{h}{6}(B + B' + 4m)$, derive the formula for the volume of (a) a prism, (b) a pyramid, (c) a sphere. [3, 4, 3]

[Suggestion: Consider each solid a special case of the prismatoid.]

* This question is based on one of the optional topics in the syllabus.