The University of the State of New York

257TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Friday, June 23, 1933 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Answer all questions in part I; in part II, answer three questions from group I and two questions from group II.

Part I is to be done first and the maximum time to be allowed for this part is one and one half hours. Merely place the answer to each question in the space provided; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one and one half hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one and one half hour period.

Group I

Answer all questions in this group. Each correct answer will receive 2½ credits. No pa credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–11) — Write on the dotted line at the right of each question expression which when inserted in the corresponding blank will make the statement true.

1 Given any two lines not in the same plane; then one and only one plane can always be passed through one of these lines ... to the other. *Ans*.....

2 Two intersecting planes each tangent to a cylinder intersect in a line which is parallel to every ... of the cylinder. *Ans*.....

3 Two acute dihedral angles whose faces are parallel are .... *Ans*.....

4 The lateral area of a prism is equal to the product of a lateral edge and the perimeter of .... *Ans*.....

5 If the slant height of a frustum of a right circular cone is 6 inches and the radii of its bases are 4 inches and 2 inches, the lateral area of the frustum is ... square inches. [Answer may be left in terms of π.] *Ans*.....

6 Two similar polyhedrons have corresponding diagonals which are in the ratio 2:3. If the total surface of the smaller polyhedron is 40 square inches, the total surface of the larger is ... square inches. *Ans*.....

7 If the sides of a spheric triangle contain 100°, 85° and 90°, the number of degrees in the largest angle of its polar triangle is .... *Ans*.....

8 The total surface of a cube is equal to exactly ... times the square of a diagonal of the cube. *Ans*.....

9 The base of a pyramid is an equilateral triangle of side a and the height of the pyramid is equal to a base edge. The volume of the pyramid expressed as a function of a is $V = \ldots$. *Ans*.....

10 If a cone of revolution and a cylinder of revolution have equal heights and equal volumes, the radius of the base of the cone is to the radius of the base of the cylinder as $\sqrt{3}$ is to .... *Ans*.....

11 A plane passes through the center of two concentric spheres; the locus of points in the given plane and equidistant from the two spheres is a .... *Ans*.....

Directions (questions 12–15) — Indicate the correct answer to each of the following questions b writing on the dotted line at the right the letter a, b or c.

12 A lune whose angle is A is drawn on a sphere whose radius is r. If A is doubled, the area of the lune is multiplied by (a) 2, (b) 4 or (c) 8. *Ans*.....

13 An exterior angle of a spheric triangle is (a) equal to, (b) less than or (c) greater than, the sum of the two nonadjacent interior angles. *Ans*.....

[OVER

14 The approximate length of the radius of the earth is 4000 miles; the approximate area of the earth bounded by the meridians 10° West and 28° West is ... square miles. [Answer may be left in terms of π.] *Ans*.....

15 The plane 5 inches from the center of a sphere of radius 5 inches is ... to the sphere. *Ans*.....

Directions (questions 16–20) — Indicate in the space at the right of each statement whether it is true or false.

16 Two lines always determine a plane. *Ans*.....

17 Two planes perpendicular to the same line are parallel. *Ans*.....

18 If the projections of two lines on a plane are parallel, the lines are always parallel. *Ans*.....

19 Two elements of a cylindric surface may not be coplanar. *Ans*.....

20 A spheric triangle whose angles are 90°, 30° and 90° covers $\frac{1}{12}$ of the sphere. *Ans*.....

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Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry.

The minimum time requirement is five recitations a week for half a school year.

Name the author of the textbook you have used in solid geometry.

Part II

Answer five questions from part II, including three questions from group I and two questions from group II.

Group I

Answer three questions from this group.

21 Prove that the volume of a triangular prism is equal to the product of its base and its altitude. [10]

22 If two planes are perpendicular to each other, a line drawn in one of them perpendicular to their intersection is perpendicular to the other. [10]

23 Describe fully, in *each* of the following, the locus of points that satisfies the given conditions: [Neither proofs nor drawings required]

a At a given distance from a given line of indefinite length. [2]

b Equidistant from three points not in one straight line. [2]

c Equidistant from two intersecting planes. [2]

d At a given distance m from a given plane and at a given distance n from a given line of indefinite length perpendicular to the plane. [4]

24 $ABCD$ is a rectangle. PA is perpendicular to the plane of the rectangle and line PB is drawn. Prove that angle PBC is a right angle. [10]

25 A certain prism has a quadrilateral for its base; prove that if the four diagonals of this prism pass through a common point, the prism is a parallelepiped. [10]

Group II

Answer two questions from this group.

Leave all work on the paper; merely writing the answers is not sufficient. Use $\pi = \frac{22}{7}$ *unless otherwise stated.*

26 The altitude of a right circular cylinder is 1 inch and the radius of the base is 8 inches. Compare the altitude of the cylinder with the altitude of a right circular cone that has the same base and the same total surface as the cylinder. [10]

27 What is the area of that portion of a sphere 10 inches in diameter which is visible from a point 15 inches from the center of the sphere? [10]

28 The slant height of a regular square pyramid is 8 inches and a dihedral angle formed by one face and the base is 70°.

a Find to the *nearest tenth* one side of the base and the altitude of the pyramid. [3, 3]

b Find the lateral area. [2]

c Find the volume. [2]

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