The University of the State of New York

251st High School Examination

SOLID GEOMETRY

Friday, June 19, 1931 — 9.15 a. m. to 12.15 p. m., only

Instructions

Do not open this sheet until the signal is given.

Answer all questions in part I; in part II, answer three questions from group I and two questions from group II.

Part I is to be done first and the maximum time to be allowed for this part is one hour. Merely place the answer to each question in the space provided; no work need be shown.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since *no credit will be given any answer in part I which is not correct and in its simplest form.*

When the signal to stop is given at the close of the one hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

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Fill in the following lines:

Name of school.....Name of pupil.....

Detach this sheet and hand it in at the close of the one hour period.

Part 1

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions (questions 1–10) — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1 The lateral area of a prism is equal to the product of a lateral edge and the perimeter of . . . . *Ans*.....

2 A regular octahedron has . . . vertices. *Ans*.....

3 A spheric triangle has three right angles and its area is 10 square inches; hence the area of the sphere must be . . . square inches. *Ans*.....

4 Two face angles of a trihedral angle are 150° and 120°. The third face angle must be between . . . degrees and . . . degrees, and may have any value between these limits. *Ans*.....

5 If the area of a sphere is 72 square inches, then the area of a lune whose angle is 40° on the sphere is . . . square inches. *Ans*.....

6 The volume of a sphere is equal to its area multiplied by . . . . *Ans*.....

7 The locus of the centers of all spheres passing through three given points is a . . . . *Ans*.....

8 If the point P is 5 inches from the plane m, the locus of all points in m at a distance of 8 inches from P is a . . . . *Ans*.....

9 The sides a and b of a spheric triangle are each a quadrant in length; therefore vertex C' of the corresponding polar triangle coincides with . . . . *Ans*.....

10 All the lines tangent to a sphere from an exterior point P form a . . . surface. *Ans*.....

Directions (questions 11–20) — Write on the dotted line at the right of each statement the word *true* or *false* to indicate whether the corresponding statement is true or false.

11 A diagonal of a cube makes an angle of 45° with each of the edges which it meets at a vertex. *Ans*.....

12 Through a given point outside a plane, one and only one line can be passed parallel to the plane. *Ans*.....

13 Every section of a cone of revolution made by a plane cutting all the elements is a circle. *Ans*.....

14 The lateral area of a regular pyramid is equal to one half the product of its altitude and the perimeter of its base. *Ans*.....

15 If the line l lies in the plane m, and the line s in the plane n, and l is perpendicular to s, then the plane m must be perpendicular to the plane n. *Ans.*.....

16 If the planes r and s are perpendicular to each other, and the line l lies in the plane r, then l must be perpendicular to the plane s. *Ans.*.....

17 A plane parallel to the base of a pyramid and halfway between the base and the vertex cuts the pyramid in a section whose area is one half that of the base. *Ans.*.....

18 A plane passed through two diagonally opposite edges of a parallelepiped divides it into two congruent triangular prisms. *Ans.*.....

19 The volume of any parallelepiped is equal to the product of the three edges which meet at a vertex. *Ans.*.....

20 If the radius of the base of a cone is doubled and its altitude halved, the volume is not changed. *Ans.*.....

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Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry.

The minimum time requirement is five recitations a week for half a school year.

Name the author of the textbook you have used in solid geometry.

Part II

Answer five questions from part II, including three questions from group I and two questions from group II.

Group I

Answer three questions from this group.

21 Prove that if two lines are cut by three parallel planes, the corresponding segments are proportional. [12]

22 Prove that the volume of a triangular pyramid is equal to one third the product of its base and its altitude. [12]

23 Prove that the sum of the interior angles of a spheric quadrilateral is greater than 360° and less than 720°. [12]

24 *a* If m and n are perpendicular planes and A a point in their line of intersection, what is the locus of the point P under *each* of the following conditions: (1) P is 3 inches from m, (2) P is 4 inches from n, (3) P is 6 inches from A? [3, 3, 3]

b How many points are there which satisfy all three conditions? [3]

Group II

Answer two questions from this group.

Leave all work on the paper; merely writing the answers is not sufficient. Use $\pi = \frac{22}{7}$ unless otherwise stated.

25 The base of a regular pyramid is a square 6 inches on each side and the four lateral faces are equilateral triangles; find to the *nearest inch* the altitude of the pyramid. [12]

26 A cylinder of revolution and a cone of revolution have bases of the same size and a common altitude of 8 inches; the lateral area of the cylinder is exactly equal to the lateral area of the cone. Find the radius of the base. [12] [Leave answer in radical form.]

27 If the earth is a sphere of radius 4000 miles, what is the area inclosed by the equator, the parallel of latitude 30° north, and the meridians of longitude 75° west and 105° west? [12]