SOLID GEOMETRY

Instructions

Do not open this sheet until the signal is given.

Answer all questions in part I; in part II, answer three questions from group I and two questions from group II.

Part I is to be done first and the maximum time to be allowed for this part is one hour.

If you finish part I before the signal to stop is given you may begin part II. However, it is advisable to look your work over carefully before proceeding to part II, since no credit will be given any answer in part I which is not correct and in its simplest form.

When the signal to stop is given at the close of the one hour period, work on part I must cease and this sheet of the question paper must be detached. The sheets will then be collected and you should continue with the remainder of the examination.

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Friday, June 21, 1929

Fill in the following lines:

Name of school...........................Name of pupil..........................

Detach this sheet and hand it in at the close of the one hour period.

Part I

Answer all questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Each answer must be reduced to its simplest form.

Directions — Write on the dotted line at the right of each question the expression which when inserted in the corresponding blank will make the statement true.

1 Two ... perpendicular to the same plane are parallel.

Ans............

2 If a line is ... to a plane, every plane passed through the line is ... to the given plane.

Ans............

3 The locus of points equidistant from two given points is the ... perpendicular to the line joining them at its mid-point.

Ans............

4 Is a right parallelepiped always a rectangular parallelepiped? [Answer *yes* or *no*.]

Ans............

5 A right section of a prism is a regular hexagon. If the lateral area is 228 and one lateral edge of the prism is 12, then one side of the hexagon is ....

Ans............

6 The area of the base of a pyramid is 12. The area of the section made by a plane parallel to the base and bisecting the altitude is ....

Ans............

7 Each lateral edge of a regular square pyramid is 5 and each side of the base is 6. The lateral area of the pyramid is ....

Ans............

8 The lateral area of a cylinder of revolution whose altitude is equal to a diameter of the base is exactly ... of its total area.

Ans............

9 The formula for the volume of a circular cone is ....

Ans............

10 The intersection of two spheres is a ....

Ans............

11 If the three sides of a spheric triangle are 60°, 80° and 50°, the spheric excess of the polar triangle is ... degrees.

Ans............

12 The sum of the angles of a spheric triangle is less than ....

Ans............

13 If the area of the surface of a sphere is 36π square feet, then its volume in terms of π is ....

Ans............

14 The area of a lune of 36° on a sphere whose radius is 14 is ....   [Use $\pi = \frac{22}{7}$]

Ans............

Questions 15–20 are based on the figure below. *AK* is a cube whose edge is 6 and lines are drawn as indicated in the figure.

15 The volume of pyramid *F–AED* is ....

16 Angle *DAF* is an angle of ... degrees.

17 The projection of line *DF* on plane *AC* is the line ....

18 The square of *DF* is exactly ... times the square of *AB*.

19 If each edge of *AK* is doubled, the volume of *AK* is multiplied by ....

20 Plane angle *AFC* is ... than a right angle.
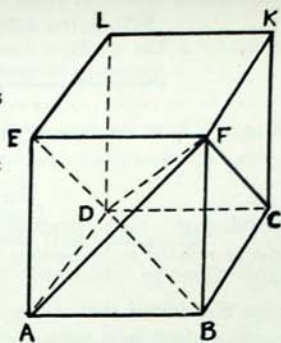
Ans............

Ans............

Ans............

Ans............

Ans............

Ans............

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Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry.

The minimum time requirement is five recitations a week for half a school year.

<u>Name the author of the textbook you have used in solid geometry.</u>

Part II
Answer five questions from part II, including three questions from group I and two questions from group II.

Group I
Answer three questions from this group.

21 Prove that the plane passed through two diagonally opposite edges of a parallelepiped divides it into two equal triangular prisms. [12]

22 Prove that if parallel planes intersect a sphere, the circles formed have the same poles. [12]

23 Prove that the equal sides of an isosceles triangle make equal angles with any plane containing the base. [12]

24 The locus of all points 3 inches from a given straight line *AB* and 5 inches from point *P* in line *AB* is to be determined.

   a What kind of geometric figure is the required locus? [6]

   b Make a drawing showing *each* locus involved. [6]

25 Given a sphere and a straight line not intersecting the sphere; it is required to construct through the line a plane tangent to the sphere. Make a drawing and give a full description of the method by which this construction is made. [3, 9]

Group II
Answer two questions from this group.
Irrational results may be left in the form of π and radicals unless otherwise stated.

26 If the lateral surface of a right circular cone is cut along an element and the surface spread out on a plane, the resulting figure is a quadrant of a circle whose radius is 6; find the volume of the cone to the *nearest tenth*. [Use $\pi = 3.14$] [12]

27 A regular hexagon, each of whose sides is 4 inches, is inscribed in a circle and the whole figure is rotated through $360°$ about one of the diagonals of the hexagon which is a diameter of the circle. Show that the difference between the areas of the surfaces generated by the circle and by the hexagon is $32\pi(2 - \sqrt{3})$ [12]

28 A machinist turns up three castings of the same kind of metal, each being a cylinder of revolution. The ratios of their altitudes are as $1:2:3$, and the ratios of their diameters are as $3:2:1$. The weight of the first casting is known to be 180 pounds; find the weight of each of the other two castings. [12]