The University of the State of New York

239TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Friday, June 17, 1927 — 9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry.

The minimum time requirement is five recitations a week for half a school year, or the equivalent.

Name the author of the textbook you have used in your study of solid geometry.

Answer eight questions, including not more than four from group I.

Group I

Do not answer more than four questions from this group.

1 Prove that if two straight lines are cut by three parallel planes, the corresponding segments are proportional. [12½]

2 Prove that if two planes are parallel, a straight line perpendicular to one of the planes is perpendicular to the other. [12½]

3 Prove that if two planes are perpendicular to each other, a straight line drawn in one of them, perpendicular to their intersection, is perpendicular to the other. [12½]

4 Prove that the volume of any pyramid is equal to one third of the product of its base and its altitude. [12½]

5 Prove that if the first of two spheric triangles is the polar triangle of the second, then reciprocally, the second is the polar triangle of the first. [12½]

Group II

Irrational results may be left in the form of π and radicals unless otherwise stated.

6 What is the locus of a point equidistant from two parallel lines? [12½]

7 The diameter of a preserving kettle is three times the diameter of a glass jar, both kettle and jar being cylindric in shape. If the depth of preserves in the kettle is two thirds the height of the jar, how many jars will be needed to hold the preserves? [12½]

8 Two planes intersect in a line AB. A third plane RS cuts the two given planes in parallel lines. Prove that RS is parallel to AB. [12½]

9 A circle and an equilateral triangle inscribed in it are revolved about an altitude of the triangle as an axis. If the altitude of the triangle is 6 inches, find the volume of the sphere and the total area of the cone generated. [5, 7½]

10 A side of the base of a regular hexagonal pyramid is 6 inches and a lateral edge is 10 inches; find the volume and the lateral area of the pyramid. [6½, 6]

11 A cylindric hole, 6 inches in diameter, is bored through a sphere, the axis of the cylinder passing through the center of the sphere. The height of the cylindric hole is 8 inches. Find the area of the surface of the sphere that remains. [12½]

12 State whether *each* of the following statements is true or false: [Write the letters *a*, *b*, *c*, *d*, *e* in a column and then write the word *true* or *false* after each letter.] [12½]

a Two lines parallel to the same plane are parallel to each other.

b Two lines perpendicular to the same line are parallel to each other.

c The lateral area of a right circular cylinder circumscribed about a sphere is equal to the area of the sphere.

d Six congruent equilateral triangles may be so placed that they completely bound a solid.

e If two circular cones have congruent bases and equal altitudes, their lateral areas are equal.