The University of the State of New York

221ST HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Monday, June 16, 1919—9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry. The minimum time requirement is two recitations a week for a school year or four recitations a week for half a school year.

Name the author of the textbook you have used in your study of solid geometry.

Answer eight questions, including three from group I, two from group II and three from group III.

Group I

Answer three questions from this group.

1 Prove that parallel lines included between parallel planes are equal.

2 Prove that if two planes are perpendicular to each other, a line in one of them perpendicular to their intersection is perpendicular to the other plane.

3 Prove that two right prisms are equal if they have equal bases and equal altitudes.

4 Prove that the volume of a triangular pyramid is equal to one third the product of its base and altitude.

5 Prove that two mutually equilateral spherical triangles on equal spheres are mutually equiangular and hence equal or symmetrical.

6 Prove that the area generated by a line revolving about an axis in its plane, not perpendicular to it, is equal to the length of the projection of the line on its axis multiplied by the circumference of a circle whose radius is perpendicular to the given line at its middle point and terminated by the axis. [Case II only, when the given line meets the axis.]

Group II

Answer two questions from this group.

7 Prove that if two equal oblique lines are parallel and meet a plane, their projections on the plane are equal.

8 Prove that the section formed by a plane parallel to two non-adjacent edges of a tetrahedron is a parallelogram.

9 Prove that if the diagonals of a quadrangular prism pass through a common point, the prism is a parallelepiped.

10 By means of the prismatoid formula, derive the formula for the volume of a frustum of a pyramid. [No authorities (reasons) required.]

Group III

Answer three questions from this group. No authorities (reasons) are required in the solutions of this group.

11 Derive the formulas for the total surface (T), the altitude (H) and the volume (V) of a regular tetrahedron whose edge is a.

12 The sides of a plane triangle are 10, 17 and 21. Find the surface and the volume of the solid generated by revolving the triangle about the side 21 as an axis.

13 A right circular cone, a right circular cylinder and a sphere have each a radius R and an altitude $2R$. Find the ratio of the sum of the total surfaces of the cone and cylinder to the surface of the sphere.

14 A right prism has a rhombus (equilateral parallelogram) for its base. If the diagonals of this base are 12 feet and 16 feet, and the height of the prism is 20 feet, find the total surface and the volume of the prism.

15 A spheric pyramid has for its base a spheric quadrilateral whose angles are 68° 24′, 149° 50′, 96° 54′ and 136° 52′. Find the volume of the pyramid if the radius of the sphere is 14.