The University of the State of New York

219TH HIGH SCHOOL EXAMINATION

SOLID GEOMETRY

Monday, June 17, 1918—9.15 a. m. to 12.15 p. m., only

Write at top of first page of answer paper (*a*) name of school where you have studied, (*b*) number of weeks and recitations a week in solid geometry. The minimum time requirement is two recitations a week for a school year or four recitations a week for half a school year.

Name the author of the textbook you have used in your study of solid geometry.

Answer eight questions, including four from group I and four from group II.

Group I

Answer four questions from this group.

1 Prove that the perpendicular is the shortest line that can be drawn from a point to a plane.

2 Prove that the intersections of two parallel planes with any third plane are parallel.

3 Prove that if a straight line is perpendicular to a plane, every plane passed through the line is perpendicular to the given plane.

4 Prove that the plane passed through two diagonally opposite edges of a parallelepiped divides it into two equivalent triangular prisms.

5 Prove that all the points of a circle on a sphere are equally distant from a pole of that circle.

6 Prove that the intersection of two spherical surfaces is a circle whose plane is perpendicular to the line which joins the centers of the spheres and whose center is in that line.

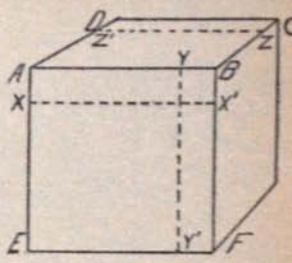
Group II

Answer four questions from this group.

7 Find the point in a plane to which lines may be drawn from two given external points on the same side of the plane so that their sum shall be the least possible.

8 If oranges 3″ in diameter sell for 30¢ a dozen and oranges 4″ in diameter sell for 50¢ a dozen, which are the cheaper by volume?

9 In the accompanying figure, $ABCDEF$ represents a solid cube. The side of this cube is $p+q$ inches. The lines xx', yy', zz' are parallel to the edges.



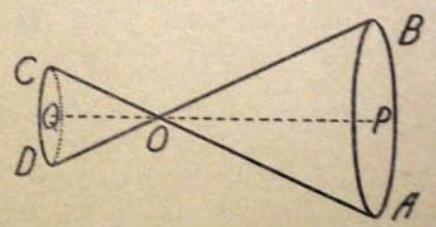
$Ax=By=Cz=q$ inches, so that $Ex=Ay=Bz=p$ inches. Saw cuts are made through the cube horizontally along xx' and vertically along yy' and zz', parallel to the faces.

a How many pieces are thus formed?

b Write the expression for the volume of each of these pieces.

c What algebraic identity results when the sum of all these volumes is set equal to the volume of the cube?

10 The following figure represents two right circular cones with a common vertex and a common axis; if $PO=2OQ$ prove that the total volume of the figure is equal to the volume of a cylinder having POQ as an axis and the smaller section as a base.



11 A cone is inscribed in a regular triangular pyramid of altitude a inches and base b square inches. Find the number of cubic inches between the surfaces of the two solids.

12 Compute the number of square inches in the area of a triangle drawn on a sphere whose diameter is 14 inches, the angles of the triangle being 84° 28′, 115° 21′ and 142° 11′.