

## High School Department

157TH EXAMINATION

## SOLID GEOMETRY

Friday, January 27, 1899—1.15 to 4.15 p. m., only

Answer 10 questions but no more. If more than 10 are answered only the first 10 answers will be considered. Draw carefully and neatly each figure in construction or proof, using letters instead of numerals. Arrange work logically. Each complete answer will receive 10 credits. Papers entitled to 75 or more credits will be accepted.

- 1 Define *pyramid, regular polyedron, generatrix, cylinder of revolution, great circle.*
- 2 Prove that through a given point in a given plane one perpendicular to that plane can be drawn and only one.
- 3 Prove that if two straight lines are intersected by three parallel planes their corresponding segments are proportional.
- 4 Prove that two dihedral angles have the same ratio as their plane angles, when the plane angles are commensurable.
- 5 Prove that every point in a plane which bisects a dihedral angle is equidistant from the faces of the angle.
- 6 Prove that the sections of a prism made by parallel planes are equal-polygons.
- 7 To what is the volume of a cone equal? Give proof.
- 8 The base of a pyramid 15 feet high contains 169 square feet; find the distance from the vertex at which a plane must be passed parallel to the base so that the section may contain 81 square feet.
- 9 Find the volume of a regular pyramid whose height is 12 feet and whose base is an equilateral triangle each side of which is 5 feet.
- 10 The height of a right circular cone is 18 inches and the radius of its base is 3 inches; find the volume of the frustum of this cone made by a plane parallel to the base and 6 inches from it.
- 11 The diameter of a sphere is 6 decimeters; find the area of a section made by a plane whose distance from the center of the sphere is 18 centimeters.
- 12 Find the total interior surface of a tunnel 5000 feet long whose section is a semicircle with a radius of 10 feet.
- 13 A light is placed 3 feet from the center of a sphere whose radius is 1 foot; find in inches the illuminated surface of the sphere.
- 14 Prove that the square of a diagonal of a rectangular parallelepiped is equal to the sum of the squares of its three dimensions.
- 15  $A$  is the vertex and  $BCD$  the base of a tetrahedron; prove that the section of the tetrahedron made by a plane parallel to both  $AB$  and  $CD$  is a parallelogram.